HARDY ALGEBRAS, BEREZIN TRANSFORMS, AND TAYLOR'S TAYLOR SERIES

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Abstract. Let $H^\infty(E)$ be the Hardy algebra of a countably generated W^*-correspondence E over a W^*-algebra with separable predual M. Also let Σ be an additive subcategory of the category of normal representations of M on separable Hilbert space. For $\sigma \in \Sigma$, $\mathbb{D}(0,1,\sigma)$ denotes the open unit ball in the intertwiner space $\mathcal{I}(\sigma^E \circ \varphi, \sigma)$, where σ^E is the representation induced by E in the sense of Rieffel and φ gives the left action of M on E. The families $\{\mathbb{D}(0,1,\sigma)\}_{\sigma\in\Sigma}$ are variants of the matricial domains first studied by Taylor in the early 1970's and more recently by Voiculescu; Popescu; Helton, Klep, McCullough, and Slinglend; and by Kalyuzhny-Verbovetski and Vinnikov. Among other things they satisfy the inclusion $\mathbb{D}(0,1,\sigma)\oplus\mathbb{D}(0,1,\tau) \subseteq \mathbb{D}(0,1,\sigma\oplus\tau)$. Each $F \in H^\infty(E)$ determines a natural, holomorphic, $B(H_\sigma)$-valued function $\widehat{F}_\sigma$ on $\mathbb{D}(0,1,\sigma)$ that we call the σ-Berezin transform of F. The family $\{\widehat{F}_\sigma\}_{\sigma\in\Sigma}$ is uniformly bounded by $\|F\|$ and satisfies the intertwining equation $C\widehat{F}_\sigma(\mathfrak{z}) = \widehat{F}_\tau(\mathfrak{w})C$ for each C that intertwines σ and τ and satisfies $C\mathfrak{z} = \mathfrak{w}(I_E \otimes C)$. Thus $\{\widehat{F}_\sigma\}_{\sigma\in\Sigma}$ satisfies variants of the intertwining relations studied by Taylor and the others cited. We show, conversely, that if $\{f_\sigma\}_{\sigma\in\Sigma}$ is a uniformly bounded family of functions, $f_\sigma : \mathbb{D}(0,1,\sigma) \to B(H_\sigma)$, such that $Cf_\sigma(\mathfrak{z}) = f_\tau(\mathfrak{w})C$, for all C that intertwine σ and τ and satisfy $C\mathfrak{z} = \mathfrak{w}(I_E\otimes C)$, then f_σ admits one of Taylor's Taylor series. We use this series to show that given $\epsilon > 0$ and R, $0 < R < 1$, there is an $F \in H^\infty(E)$ that is a finite sum of tensors such that $\|f_\sigma(\mathfrak{z}) - \widehat{F}_\sigma(\mathfrak{z})\| < \epsilon$ for all σ and for all $\mathfrak{z} \in \mathbb{D}(0,R,\sigma)$.