Automata and Formal Languages

Review Questions

1. Let Σ be an alphabet containing the letter a. Let $f : \Sigma^* \longrightarrow \Sigma^*$ be the function which, given any word $w \in \Sigma^*$, replaces all letters in the even locations by a. That is, if $w = \sigma_1\sigma_2 \dots \sigma_n$, then $f(w) = \sigma'_1\sigma'_2 \dots \sigma'_n$, where $\sigma'_i = \sigma_i$ for odd i and $\sigma'_i = a$ for even i.

   (a) Show that, if L is a regular language, then so is $f(L)$ in the following two ways:

      i. directly from the definition of regular languages. (Hint: You may use the fact that, if L is a regular language, then so are the languages L_{even} and L_{odd}, consisting of all words of even and of odd length in L, respectively.)

      ii. by constructing, given a finite automaton M accepting the language L, another finite automaton M' accepting $f(L)$. Define M' precisely, give a short explanation, and show a (not too trivial) example of an automaton M and the corresponding M'.

   (b) Suppose $f(L)$ is regular. Is L necessarily regular as well?

   (c) Show that, if L is a context-free language, then so is $f(L)$, by showing how, given a context-free grammar G accepting L, you can construct another context-free grammar G' accepting $f(L)$. Give an example of your construction.

   (d) Provide another proof of the first part of the question using grammars.

   (e) Provide another proof of part (c) using pushdown automata.

   (f) Construct a Turing machine computing the function f.

(g) Characterize the language $f(\Sigma^*)$ and construct a Turing machine deciding it.

2. Prove or disprove:

   (a) If L is a regular language, then every sufficiently long word $w \in L$ may be written in the form $w = uvxyz$ with $uv^m xy^n z \in L$ for every $m, n \geq 0$.

   (b) The language $L \subseteq \{a, b, c\}^*$, consisting of all words w for which $|w|_a = |w|_b = |w|_c$ and $|u|_a \geq |u|_b \geq |u|_c$ for each prefix u of w, is context-free.

   (c) Given a deterministic finite automaton $M = (Q, \Sigma, \delta, s, A)$, there exists a constant $l = l(|Q|, |\Sigma|)$, such that the language $L(M)$ is accepted by a regular expression of length not exceeding l. Moreover, it is possible to find an explicit upper bound on the required l (in terms of $|Q|$ and $|\Sigma|$).