BEN GURION UNIVERSITY  אוניברסיטת בן גוריון בנגב

Prof. Amnon Yekutieli
Department of Mathematics
Ben Gurion University
Be'er Sheva 84105, ISRAEL
Email: amyekut@math.bgu.ac.il
Web: www.math.bgu.ac.il/~amyekut

12 February 2016

Derived Categories II

Course in 2nd Semester 2015-16

Audience & Prerequisites. This is an advanced course, aimed at M.Sc. and Ph.D. students, post-docs and researchers. *Participants from outside the BGU community are welcome*. The lectures will be in English.

The course is a continuation of *Derived Categories I* that was given in the 1st semester. It is recommended that participants shall have a reasonable knowledge of the material covered in the first course (see below). For some of the examples and applications, a familiarity with commutative algebra, ring theory, algebraic geometry and algebraic topology will be helpful.

Organization. The course will meet once a week for a 2 hour lecture.
Time: Wednesday 12:00-14:00
Location: building 58 room 201
First Meeting: 9 March 2016
Course web page: AY web page | teaching | Derived Categories II

Potential participants are urged to get in touch with me regarding information about registration. The course notes are likely to be published as a textbook.

Note that the BGU AG&NT Seminar meets on Wednesdays at 15:00.

On the subject. See the announcement of the 1st course, located at: AY web page | teaching | Derived Categories I.

What we did in the first course. Besides a review of categories (in general and abelian), most of the effort was the study of *DG categories*. We introduced the DG category $\mathsf{C}(A, \mathsf{M})$ of DG A-modules in M. Here A is a DG ring, and M is an abelian category. This new framework includes in it both the category of complexes over M, and the category of DG A-modules. The homotopy category $\mathsf{K}(A, \mathsf{M})$ is triangulated. One of the main results we proved is that any DG functor

$$F : \mathsf{C}(A, \mathsf{M}) \to \mathsf{C}(B, \mathsf{N})$$

(regardless of its source) induces a triangulated functor

$$F : \mathsf{K}(A, \mathsf{M}) \to \mathsf{K}(B, \mathsf{N}).$$

This is a *new theorem*, and the participants of the course contributed to the formulation and the proof.

Complete notes of the first course are available on its web page – it is the file "course-notes_public-18.pdf" there.

Topics for the second course. Here is a tentative list of topics. Again, the plan is to talk about some established results (both from textbooks and research papers), and also, if and when it becomes appropriate, to make some new progress in the theory.

Topics (1-3) below are crucial, and they are expected to occupy less than half the course. The remaining topics (including their order) will be chosen together with the participants. (It is conceivable that there will be a third part to the course.)

(1) Localization of categories, and the derived category $\mathsf{D}(A, \mathsf{M})$ of DG A-modules in M.
(2) Left and right derived functors.
(3) Resolutions of DG modules (K-projective, K-injective and K-flat resolutions).
(4) Commutative algebra via derived categories. Dualizing complexes, local duality, MGM equivalence, rigid dualizing complexes.
(5) Geometric derived categories (of sheaves on spaces). Direct and inverse image functors, Grothendieck duality, Poicaré-Verdier duality, perverse sheaves.
(6) Derived categories associated to noncommutative rings. Dualizing complexes, tilting complexes, the derived Picard group, derived Morita theory.
(7) Survey of infinity categories. Survey of derived categories in modern algebraic geometry and mathematical physics. Survey of derived algebraic geometry.