Title: Sampling a random field along a stationary process, related questions in ergodic theory

Abstract

For a sequence (X_k) of real iid random variables with common probability distribution function F, the empirical process is defined by $W_n(s) := \sum_{k=0}^{n-1} [\mathbf{1}_{X_k \leq s} - F(s)]$.

Among classical results on this process, the following are well known.
- the Glivenko-Cantelli theorem: $\sup_s \frac{1}{n}|W_n(s)| \to 0$;
- a functional central limit theorem for $W_n(s)$ after normalisation.

We will present extensions of these results when the process is sampled along a sequence of times (z_n), in particular when (z_n) is a sequence of ergodic sums generated by a dynamical system.

More precisely, if (X, μ, T) is a dynamical system and $f : X \to \mathbb{Z}^d$, we take, for $x \in X$, $z_n = S_n(x) := \sum_{k=0}^{n-1} f(T^k x), n \geq 1$. Then, if $(X_{\underline{\ell}}, \underline{\ell} \in \mathbb{Z}^d)$ is a random field indexed by $\mathbb{Z}^d$ of real iid r.v.s (or more generally of associated r.v.s), we study the sampled process $(X_{S_n(x)})_{n \geq 1}$.

To apply general criteria, a precise information about the number of visits of S_n to a site before time n is needed. We will give examples where we can conclude. When (S_n) is a random walk, this is related to limit theorems in random scenery and to results (going back to Erdös and several authors) on limit laws for the "maximal multiplicity" in n steps of a random walk.

This is a joint work with Guy Cohen (Ben-Gurion University).