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פרופ' אמנון יקותיאלי
המחלקה למתמטיקה
אוניברסיטת בן גוריון
באר שבע 84105

5 October 2025

Graduate Course

Modern Algebraic Geometry 1

Semester A (Fall) 2025-26

Catalog no: 201.2.2131

General Description. Algebraic geometry (AG) is the study of geometric objects, called *algebraic varieties,* defined by *polynomial equations* (e.g. circles and ellipses). Since ancient times mathematicians were interested in the size and shape of the solution sets of polynomial equations, classifying them, learning how they intersect, etc. During the 19th century AG became intertwined with *complex analytic geometry* and with *number theory*, and later also with *algebraic topology*. A giant leap in AG occurred early in the 20th century, with the emergence of *commutative algebra* as the basic tool for treating algebraic varieties. Around 1960 there was another revolution in AG: the introduction (by Grothendieck) of *schemes*, which are algebro-geometric objects that are much more general than varieties, and can very effectively describe and analyze questions arising from number theory, and also permit an algebro-geometric version of differential calculus. In the 21st century we see further progress in the foundations of AG: it is merging with modern algebraic topology, and goes under the name *derived algebraic geometry*. There is also a theory of *derived commutative algebra*.

Besides being a very interesting and vast mathematical theory on its own, AG is essential, as a working tool, for several other mathematical areas. Among them are *representations of groups and Lie algebras* (currently the Langlands program is the most active research in this direction), *number theory* (notably the proof by Wiles in the 1990s of the Fermat Conjecture, which relied mainly on the study of algebro-geometric objects called elliptic curves), *homotopy theory* (e.g. chromatic homotopy, which also involves elliptic curves), and *mathematical physics* (mirror symmetry, string theory, quantum field theories, mathematical Feynman integrals, much due to Kontsevich). In conjunction with the development of AG, there was rapid development of *homological algebra*, including the theory of *derived categories* (by Grothendieck and his students). Lastly, from a general pedagogical viewpoint, a working knowledge of AG is useful for gaining a deeper understanding of a multitude of other mathematical areas.

This is a course on *algebraic varieties over an algebraically closed field*. Technically it is much easier than a course on schemes, but it does not touch upon arithmetic geometry. **The course will continue into the second semester as Modern Algebraic Geometry 2**. We shall cover most of the standard material, with some additional glances into more advanced or specialized topics. The topics listed below will be adapted – to some extent – to the background and capabilities of the registered students.

Audience: The course is intended for graduate students at BGU. Strong undergraduate students, and outside students, can also attend.

Participation: **All interested students must contact me by email, to get permission to participate, and to obtain the Zoom link.**

Organization: The course will consist of one lecture (2 hours) per week, and homework.

Time: **Sunday 12:00 – 14:00.**

Place: **Broadcast on Zoom.**

Grades: The course grades are pass/fail. Passing requires attending all lectures and submitting almost all the homework assignments. Homework will be assigned each week.

Course web page:
`https://sites.google.com/view/amyekut-math/home/teaching/modern-ag-1-2025-26`

Language: English.

Prerequisite courses:
1. "Introduction to Commutative Algebra" 201.1.7071 or "Commutative Algebra" 201.2.2011.
2. "Introduction to Topology" 201.1.0091.

Recommended knowledge: Differentiable manifolds; Galois theory; algebraic topology; homological algebra; complex analysis.

Course Topics: For the two semesters, and depending on rate of progress. Many topics will only be sketched (or omitted).

1. **Projective space over the real numbers – a warm-up.** We will learn about the projective spaces $\mathbf{P}^n(\mathbb{R})$, first as topological spaces, then as differentiable manifolds, and then as ringed spaces.
2. **Ringed spaces.** Topological spaces equipped with sheaves of rings of functions. Maps between ringed spaces. Study familiar examples: topological spaces and differentiable manifolds.
3. **Categories and functors.** A systematic but gradual study, mixed in with the other topics.
4. **Recalling commutative algebra.** Noetherian rings, Hilbert Basis Theorem, algebraic and transcendental field extensions, Hilbert Nullstellensatz. Localization of rings, prime ideals, flatness. Tensor products of rings.
5. **Affine algebraic varieties.** (All varieties are over an algebraically closed field $\mathbb{K}$.) The Zariski topology and the sheaf of algebraic functions. Maps of affine varieties, fibered products, subvarieties, local rings. Dimension of an affine variety and the transcendence degree of its function field.
6. **Projective algebraic varieties.** Definition by quotients, by homogeneous coordinate rings and by gluing. Fibered products of projective varieties. Quasi-projective varieties.
7. **Algebraic varieties.** Gluing, the separation condition, definition of an algebraic variety.
8. **Types of maps between varieties**. Finite, quasi-finite, flat, affine and projective maps.
9. **Algebraic schemes** (sketch).
10. **Vector bundles**. Definition, examples. Gluing vector bundles.
11. **Sheaves of modules**. Definition and examples. From locally free sheaves to vector bundles and back. Coherent sheaves. Serre's Theorem on affine open sets.
12. **Line bundles and the Picard group** (sketch). Ample line bundles and morphisms to projective space. The automorphism group of projective space.

13. **The classification of curves** (sketch). Nonsingular projective curves and their functions fields. Elliptic curves.
14. **Enumerative geometry** (sketch). Bézout Theorem. Some intersection theory. A survey of modern methods.
15. **Birational geometry** (sketch). Blowups. Some examples. Singularities and their resolutions. A survey of modern methods.
16. **Sheaf cohomology** (sketch). Riemann-Roch Theorem for curves. A quick tour of Serre Duality and Grothendieck Duality.
17. **Algebraic groups** (sketch). Some examples. Lie algebras. Group actions, torsors, quotients.
18. **Differential algebraic geometry** (sketch). The tangent bundle, derivations, differential forms. Étale and smooth maps.

Course Notes: Typed course notes will be available every week on the course web page. These will be based partly on the books listed below, and mostly on the lecturer's preference of teaching. The homework exercises will be included in the notes.

Bibliography.

1. Hartshorne, "Algebraic Geometry", Springer, 1977.
2. Gathmann, "Algebraic Geometry" (2022), free online book.
3. Reid, "Undergraduate Algebraic Geometry", Cambridge, 1988. Online 2012.
4. Liu, "Algebraic Geometry and Arithmetic Curves", Oxford, 2002.
5. Vakil, "THE RISING SEA – Foundations of Algebraic Geometry" (2017), free online book.
6. Altman and Kleiman, "A Term of Commutative Algebra" (2021), free online book.
7. D. Eisenbud, "Commutative Algebra", GTM 150, Springer, 1995.
8. Course notes, updated weekly, on the course web page.