

Department of Mathematics, BGU

BGU Probability and Ergodic Theory (PET) seminar

On *Thursday, April ,3 2025*

At *11:10 – 12:00*

In *101-*

Aryeh Kontorovich (BGU (CS))

will talk about

On the tensorization of the variational distance

Abstract:

On the tensorization of the variational distance - PET Seminar talk

Aryeh Kontorovich (BGU, CS)

April 3, 2025

Abstract

If one seeks to estimate the total variation between two product measures $\|P_{1:n}^{\otimes} - Q_{1:n}^{\otimes}\|$ in terms of their marginal TV sequence $\delta = (\|P_1 - Q_1\|, \|P_2 - Q_2\|, \dots, \|P_n - Q_n\|)$, then trivial upper and lower bounds are provided by $\|\delta\|_{\infty} \leq \|P_{1:n}^{\otimes} - Q_{1:n}^{\otimes}\| \leq \|\delta\|_1$. We improve the lower bound to $\|\delta\|_2 \leq \|P_{1:n}^{\otimes} - Q_{1:n}^{\otimes}\|$, thereby reducing the gap between the upper and lower bounds from $\sim n$ to $\sim \sqrt{n}$. Furthermore, we show that *any* estimate on $\|P_{1:n}^{\otimes} - Q_{1:n}^{\otimes}\|$ expressed in terms of δ must necessarily exhibit a gap of $\sim \sqrt{n}$ between the upper and lower bounds in the worst case, establishing a sense in which our estimate is optimal. Finally, we identify a natural class of distributions for which $\|\delta\|_2$ approximates the TV distance up to absolute multiplicative constants.