

Department of Mathematics, BGU

AGNT

On Wednesday, November 13, 2024

At 14:10 – 15:10

In 101-

Mikhail Borovoi (TAU)

will talk about

The power operation in the Galois cohomology of a reductive group over a number field

Abstract: For a number field K admitting a real embedding, it is impossible to construct a functorial in G group structure in the Galois cohomology pointed set $H^1(K, G)$ for all connected reductive K -groups G . However, over an arbitrary number field K , we define a *diamond* (or *power*) operation of raising to power n $(x, n) \mapsto x^{\{\Diamond n\}}: H^1(K, G) \times Z \rightarrow H^1(K, G)$. We show that this operation has many functorial properties. When G is a torus, the set $H^1(K, G)$ has a natural group structure, and $x^{\{\Diamond n\}}$ coincides with the n -th power of x in this group.

For a cohomology class x in $H^1(K, G)$, we define the period $\text{per}(x)$ to be the greatest common divisor of $n > 0$ such that $x^{\{\Diamond n\}} = 1$, and the index $\text{ind}(x)$ to be the greatest common divisor of the degrees $[L:K]$ of finite separable extensions L/K splitting x . These period and index generalize the period and index of a central simple algebra over K (in the special case where G is the projective linear

group PGL_n , the elements of $H^1(K, G)$ can be represented by central simple algebras). For an arbitrary reductive group G defined over a local or global field K , we show that $\mathrm{per}(x)$ divides $\mathrm{ind}(x)$, that $\mathrm{per}(x)$ and $\mathrm{ind}(x)$ have the same prime factors, but the equality $\mathrm{per}(x)=\mathrm{ind}(x)$ may not hold.

The talk is based on a joint work with Zinovy Reichstein.