Department of Mathematics, BGU

BGU Probability and Ergodic Theory (PET) seminar

On *Thursday, December ,21 2023*

At *11:10 − 12:00*

In *101-*

Tom Meyerovitch (BGU)

will talk about

Absolute Retracts and the Map Extension Property for Multidimensional Subshfits

Abstract: Subshfits of finite type are the central objects studied in symbolic dynamics.

In the one dimensional case, (e.g. subshfits of finite type when the acting group is Z, the group of integers), although there are dffiicult standing unsolved problems (in particular, the isomorphism problem), there is a reasonable and fairly developed structure theory:

- Any Z-subshfit of finite type "decomposes" into irreducible components and wandering points, where any irreducible SFT becomes topologically mixinig after passing to some power of the shfit.

- Krieger's embedding theorem provides "essentially checkable" necessary and sufficient conditions for an arbitrary subshfit to embed in a given topologically mixing SFT.

- Boyle's factor theorems give "essentially checkable" conditions for factoring between mixing SFTs.

The situation for multidimensional subshfits is far less structured and far more mysterious.

By now it is well-known that multidimensional subshfits of finite type can exhibit a wild variety of "pathological behavior''.

One is soon faced with undecidability issues, and there seems to be little hope to obtain a tractable structure theory in complete generality.

Over the years various properties of multidimensional subshfits have been introduced and studied, in an attempt to recover and generalize some structural aspects of the one-dimensional theory for a natural class.

Among these properties: "square mixing", "block gluing", "strong irreducibility", "topological strong spatial mixing", "the finite extension property" and more...

In this talk I will introduce a natural class of multidimensional subshfits of finite type for which I have obtained extensions of the fundamental theorems from the one dimensional case.

This new class of subshfits has various equivalent characterizations. The first characterization is the map extension property of subshfits.

The map extension property has been introduced implicitly by Mike Boyle in the early 1980's for Z-subshfits.

In a suitable natural formulation, in the context of subshfits, it turns out to coincide with the notion of an absolute retract, introduced by Borsuk in the 1930's.

The map extension property is a stronger property than strong irreducibly, but it still holds for a variety of "reasonable'' subshfits such as any subshfit with a safe symbol or proper colorings of (the standard Cayley graph of) Z^2 with 5 or more colors.

A Z-subshfit has the map extension property fi and only fi it is a mixing subshfit of finite type.

The map extension property allows a meaningful complete multidimensional generalization of both Kreiger's embedding theorem and of Boyle's lower entropy factor theorem (partial generalization have been obtained in previous work for other classes).