

Department of Mathematics, BGU

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## Combinatorics Seminar

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*On Tuesday, November 20 2018*

*At 16:00 – 17:00*

*In 201*

Noam Lifshitz (Bar Ilan)

will talk about

### **Sparse sharp thresholds and hypercontractivity**

Abstract: The sharp threshold phenomenon is a central topic of research in the analysis of Boolean functions. Here, one aims to give sufficient conditions for a monotone Boolean function  $f$  to satisfy  $\mu_p(f) = o(\mu_q(f))$ , where  $q = p + o(p)$ , and  $\mu_p(f)$  is the probability that  $f=1$  on an input with independent coordinates, each taking the value 1 with probability  $p$ .

The dense regime, where  $\mu_p(f) = \Theta(1)$ , is somewhat understood due to seminal works by Bourgain, Friedgut, Hatami, and Kalai. On the other hand, the sparse regime where  $\mu_p(f) = o(1)$  was out of reach of the available methods. However, the potential power of the sparse regime was suggested by Kahn and Kalai already in 2006.

In this talk we show that if a monotone Boolean function  $f$  with  $\mu_p(f) = o(1)$  satisfies some mild pseudo-randomness conditions then it exhibits a sharp threshold in the interval  $[p, q]$ , with  $q = p + o(p)$ . More specifically, our mild pseudo-randomness hypothesis is that the  $p$ -biased measure of  $f$  does not bump up to

$\Theta(1)$  whenever we restrict  $f$  to a sub-cube of constant co-dimension, and our conclusion is that we can find  $q=p+o(p)$ , such that  $\mu_p(f)=o(\mu_q(f))$ .

**Please Note the Unusual Time and Place!**