

## The Department of Mathematics

2024–25–B term

**Course Name** Introduction to Set Theory

**Course Number** 201.1.0171

**Course web page**

<https://math.bgu.ac.il/en/teaching/spring2025/courses/intro-to-logic-and-sets>

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**Office Hours** <https://math.bgu.ac.il/en/teaching/hours>

### Abstract

### Requirements and grading<sup>1</sup>

### Course topics

- .1 Partially ordered sets. Chains and antichains. Examples. Erdos–Szekeres’ theorem or a similar theorem. The construction of a poset over the quotient space of a quasi-ordered set.
- .2 Comparison of sets. The definition of cardinality as an equivalence class over equinumerosity. The Cantor-Bernstein theorem. Cantor’s theorem on the cardinality of the power-set.
- .3 Countable sets. The square of the natural numbers. Finite sequences over a countable set. Construction of the ordered set of rational numbers. Uniqueness of the rational ordering.
- .4 Ramsey’s theorem. Applications.
- .5 The construction of the ordered real line as a quotient over Cauchy sequences of rationals.
- .6 Konig’s lemma on countably infinite trees with finite levels. Applications. A countable graph is  $k$ -colorable iff every finite subgraph of it is  $k$ -colorable.

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<sup>1</sup>Information may change during the first two weeks of the term. Please consult the webpage for updates



- .7 Well ordering. Isomorphisms between well-ordered sets. The axiom of choice formulated as the well-ordering principle. Example. Applications. An arbitrary graph is  $k$ -colorable iff every finite subgraph is  $k$ -colorable.
- .8 Zorn's lemma. Applications. Existence of a basis in a vector space. Existence of a spanning tree in an arbitrary graph.
- .9 Discussion of the axioms of set theory and the need for them. Russel's paradox. Ordinals.
- .10 Transfinite induction and recursion. Applications. Construction of a subset of the plane with exactly 2 point in every line.
- .11 Infinite cardinals as initial ordinals. Basic cardinal arithmetic. Cardinalities of well known sets. Continuous real functions, all real functions, the automorphisms of the real field (with and without order).