אוניברסיטת
בן-גוריון בנגב

The Department of Mathematics

2021–22–A term

Course Name Homological Algebra

Course Number 201.2.2091

Course web page
`https://math.bgu.ac.il//~amyekut/teaching/2021-22/homol-alg/course_page.html`

Lecturer Prof. Amnon Yekutieli, <amyekut@bgu.ac.il>, Office 202

Office Hours `https://math.bgu.ac.il/en/teaching/hours`

Abstract

Course Topics: (as much as time permits)

.1 Review of prior material. On rings, ideals and modules (including noncommutative rings).

.2 Categories and functors. Emphasis on linear categories. (This topic will be introduced gradually, as we go along.)

.3 Universal constructions. Free modules, products, direct sums, polynomial rings.

.4 Tensor products. Definition, construction and properties.

.5 Exactness. Exact sequences and functors.

.6 Special modules. Projective, injective and flat modules.

.7 Complexes of modules. Operations on complexes, homotopies, the long exact cohomology sequence.

.8 Resolutions. Projective, flat and injective resolutions.

.9 Left and right derived functors. Applications to commutative algebra.

.10 Further applications of derived functors. Classfiication problems, extensions.

.11 Morita Theory.

(Some of the material might move to the subsequent course "Commutative Algebra")

For an updated syllabus and course requirements see the course web page[1]

Requirements and grading[2]

see course web page[3]

[1] https://www.math.bgu.ac.il/~amyekut/teaching/2021-22/homol-alg/course_page.html

[2] Information may change during the first two weeks of the term. Please consult the webpage for updates

[3] https://www.math.bgu.ac.il/~amyekut/teaching/2021-22/homol-alg/course_page.html

syllabus – new plan
7 June 2021
Amnon Yekutieli

Homological Algebra

Fall Semester 2021-22

Catalog Number: 201.2.2091

Prerequities:

1. Algebraic Structures
2. Introduction to Topology

Recommended:

1. Introduction to Commutative Algebra
2. Introduction to Algebraic Geometry
3. Basic Concepts in Topology and Geometry

Course Topics: (as much as time permits)

1. **Review of prior material.** On rings, ideals and modules (including noncommutative rings).
2. **Categories and functors.** Emphasis on linear categories. (This topic will be introduced gradually, as we go along.)
3. **Universal constructions.** Free modules, products, direct sums, polynomial rings.
4. **Tensor products.** Definition, construction and properties.
5. **Exactness.** Exact sequences and functors.
6. **Special modules.** Projective, injective and flat modules.
7. **Complexes of modules.** Operations on complexes, homotopies, the long exact cohomology sequence.
8. **Resolutions.** Projective, flat and injective resolutions.
9. **Left and right derived functors.** Applications to commutative algebra.
10. **Further applications of derived functors.** Classification problems, extensions.
11. **Morita Theory.**

(Some of the material might move to the subsequenct course "Commutative Algebra")

אוניברסיטת
בן-גוריון בנגב

Course topics

.1 Recalling prior material. Rings (including noncommutative), ideals, modules and bimodules, exact sequences, infinite direct sums and products, tensor products of modules and rings.

.2 Categories and functors. Morphisms of functors, equivalences. Linear categories and linear functors. Exactness of functors.

.3 Special modules. Projective, injective and flat modules.

.4 Morita Theory. Equivalences of module categories realized as tensor products.

.5 Complexes of modules. Operations on complexes, homotopies, the long exact cohomology sequence.

.6 Resolutions. Projective, injective and flat resolutions – existence and uniqueness.

.7 Left and right derived functors. The general theory. Tor and Ext functors.

.8 Applications to commutative algebra. Some local and global theorems, involving Tor and Ext functors. Derived completion and torsion functors.

.9 Sheaf cohomology. A survey of the role of homological algebra in geometry.

.10 Nonabelian cohomology. A survey of classfiication theorems: Galois cohomology, vector bundles.

Bibliography

.1 R. Hartshorne, "Algebraic Geometry", Springer-Verlag, New-York, .1977

.2 P.J. Hilton and U. Stammbach, "A Course in Homological Algebra", Springer, .1971

.3 S. Maclane, "Homology", Springer, .1994

.4 J. Rotman, "An Introduction to Homological Algebra", Academic Press, .1979

.5 L.R. Rowen, "Ring Theory" (Student Edition), Academic Press, .1991

.6 C. Weibel, "An introduction to homological algebra", Cambridge Univ. Press, .1994

.7 M. Kashiwara and P. Schapira, Sheaves on Manfiolds, Springer, .1990

.8 The Stacks Project[4], an online reference, J.A. de Jong (Editor). (9) A. Yekutieli, "Derived Categories", Cambridge Univ. Press, .2019 Free prepublication version[5]. (10) Course notes, to be uploaded every week to the course web page

[4] http://stacks.math.columbia.edu
[5] https://arxiv.org/abs/1610.09640v4