Department of Mathematics, BGU

Colloquium

On *Tuesday, November ,4 2025*

At *14:30 – 15:30*

In *Math 101-*

Dmitry Gourevitch (Weizmann Institute)

will talk about

Orthogonal families of hypergeometric polynomials

Abstract: We consider quasi-orthogonal polynomial families - those that are orthogonal with respect to a non-degenerate bilinear form defined by a linear functional - in which the ratio of successive coefficients is given by a rational function f(n,k) which is polynomial in n. Here, n is the index of the polynomial, and k of the coefficient. We show that, up to rescaling and renormalization, there are only five such families.

More generally, we define an auxiliary basis for the space of polynomials, called Newtonian bases, and consider coefficients with respect to this basis rather than the standard monomial basis. We call the polynomial families that satisfy the rationality conditions on ratio of successive coefficients with respect to this basis HG-families. We show that, up to rescaling, shfit, and renormalization, there are only 10 quasi-orthogonal HG-families. Each family arises as a specialization of some hypergeometric series. I will define this notion in the talk. Eight

of the 10 families are classical very useful polynomial families, and we view our theorem as a classfiication result in the theory of special functions.

We also consider the more general rational HG-families, i.e. quasi-orthogonal families in which the ratio f(n,k) of successive coefficients is allowed to be rational in n as well. I will formulate the two main theorems, one on quasi-orthogonal HG-families and one on rational quasi-orthogonal HG-families, as well as the main ideas of the proofs. They are of algebraic nature.

This is a joint work with Joseph Bernstein and Siddhartha Sahi.