Department of Mathematics, BGU

BGU Probability and Ergodic Theory (PET) seminar

On *Thursday, December ,4 2025*

At *11:10 – 12:00*

In *101-*

Michael Lin

will talk about

Entropy for power-bounded linear operators and Sarnak's conjecture

Abstract:

Entropy for power-bounded linear operators and Sarnak's conjecture

Michael Lin, Ben-Gurion University

Abstract Let μ be the Möbius function, defined for $n \in \mathbb{N}$ by $\mu(1) = 1$, $\mu(n) = (-1)^n$ if n is the product of n distinct primes, and $\mu(n) = 0$ if n has a square factor. This function is important in number theory, due to its connections to the Prime Number theorem and the Riemann Hypothesis.

Sarnak conjectured that whenver X is a compact space and τ is a continuus map of X into itself with zero toplogical entropy, then

$$\frac{1}{N}\sum_{k=1}^{N} \mu(k) f(\tau^k x) \to 0, \quad \forall f \in C(X),\ \forall x \in X.$$

When we denote $Tf = f \circ \tau$ for $f \in C(X)$, the above convergence is equivalent to

$$\frac{1}{N}\sum_{k=1}^{N} \mu(k) T^k f \to 0 \text{ weakly}, \quad \forall f \in C(X). \tag{1}$$

It is our aim to study convergence like (1) for power-bounded operators on (real or complex) Banach spaces. A power-bounded T on a Banach space E becomes a contraction under the equivalent norm $|||x||| := \sup_{n\geq 0} \|T^n x\|$, so we assume T to be a contraction.

For a contraction T on E we define an operator topological entropy $h^*_{top}(T)$, and prove that if Sarnak's conjecture is true, then $h^*_{top}(T) = 0$ implies

$$\frac{1}{N}\sum_{k=1}^{N} \mu(k) T^k v \to 0 \text{ weakly}, \quad \forall v \in E.$$

For some classes of operators T, in particular contractions in reflexive Banach spaces, we show $h^*_{top}(T) = 0$, and prove directly even the strong convergence $\|\frac{1}{N}\sum_{k=1}^{N} \mu(k) T^k v\| \to 0$ for every $v \in E$.

Joint work with el Houcein el Abdalaoui (Rouen)