

Department of Mathematics, BGU

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# AGNT

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On *Wednesday, November ,13 2024*

At *14:10 – 15:10*

In *101-*

Mikhail Borovoi (TAU)

will talk about

## **The power operation in the Galois cohomology of a reductive group over a number field**

Abstract: For a number field  $K$  admitting a real embedding, it is impossible to construct a functorial in  $G$  group structure in the Galois cohomology pointed set  $H^1(K, G)$  for all connected reductive  $K$ -groups  $G$ . However, over an arbitrary number field  $K$ , we define a *diamond* (or *power*) operation of raising to power  $n$   $(x, n) \mapsto x^{\{\Diamond n\}}: H^1(K, G) \times Z \leftarrow H^1(K, G)$ . We show that this operation has many functorial properties. When  $G$  is a torus, the set  $H^1(K, G)$  has a natural group structure, and  $x^{\{\Diamond n\}}$  coincides with the  $n$ -th power of  $x$  in this group.

For a cohomology class  $x$  in  $H^1(K, G)$ , we define the period  $\text{per}(x)$  to be the greatest common divisor of  $n > 0$  such that  $x^{\{\Diamond n\}} = 1$ , and the index  $\text{ind}(x)$  to be the greatest common divisor of the degrees  $[L:K]$  of finite separable extensions  $L/K$  splitting  $x$ . These period and index generalize the period and index of a central simple algebra over  $K$  (in the special case where  $G$  is the projective linear

group  $\mathrm{PGL}_n$ , the elements of  $H^1(K, G)$  can be represented by central simple algebras). For an arbitrary reductive group  $G$  defined over a local or global field  $K$ , we show that  $\mathrm{per}(x)$  divides  $\mathrm{ind}(x)$ , that  $\mathrm{per}(x)$  and  $\mathrm{ind}(x)$  have the same prime factors, but the equality  $\mathrm{per}(x)=\mathrm{ind}(x)$  may not hold.

The talk is based on a joint work with Zinovy Reichstein.