

Department of Mathematics, BGU

BGU Probability and Ergodic Theory (PET) seminar

On Thursday, December, 21 2023

At 11:10 – 12:00

In 101-

Tom Meyerovitch (BGU)

will talk about

Absolute Retracts and the Map Extension Property for Multidimensional Subshifts

Abstract: Subshifts of finite type are the central objects studied in symbolic dynamics.

In the one dimensional case, (e.g. subshifts of finite type when the acting group is \mathbb{Z} , the group of integers), although there are difficult standing unsolved problems (in particular, the isomorphism problem), there is a reasonable and fairly developed structure theory:

- Any \mathbb{Z} -subshift of finite type “decomposes” into irreducible components and wandering points, where any irreducible SFT becomes topologically mixing after passing to some power of the shift.

- Krieger's embedding theorem provides "essentially checkable" necessary and sufficient conditions for an arbitrary subshift to embed in a given topologically mixing SFT.
- Boyle's factor theorems give "essentially checkable" conditions for factoring between mixing SFTs.

The situation for multidimensional subshifts is far less structured and far more mysterious.

By now it is well-known that multidimensional subshifts of finite type can exhibit a wild variety of "pathological behavior".

One is soon faced with undecidability issues, and there seems to be little hope to obtain a tractable structure theory in complete generality.

Over the years various properties of multidimensional subshifts have been introduced and studied, in an attempt to recover and generalize some structural aspects of the one-dimensional theory for a natural class.

Among these properties: "square mixing", "block gluing", "strong irreducibility", "topological strong spatial mixing", "the finite extension property" and more...

In this talk I will introduce a natural class of multidimensional subshifts of finite type for which I have obtained extensions of the fundamental theorems from the one dimensional case.

This new class of subshifts has various equivalent characterizations. The first characterization is the map extension property of subshifts.

The map extension property has been introduced implicitly by Mike Boyle in the early 1980's for Z -subshifts.

In a suitable natural formulation, in the context of subshifts, it turns out to coincide with the notion of an absolute retract, introduced by Borsuk in the 1930's.

The map extension property is a stronger property than strong irreducibility, but it still holds for a variety of "reasonable" subshifts such as any subshift with a safe symbol or proper colorings of (the standard Cayley graph of) Z^2 with 5 or more colors.

A Z -subshift has the map extension property if and only if it is a mixing subshift of finite type.

The map extension property allows a meaningful complete multidimensional generalization of both Kreiger's embedding theorem and of Boyle's lower entropy factor theorem (partial generalization have been obtained in previous work for other classes).