Department of Mathematics, BGU

Logic, Set theory and Topology

On *Tuesday, November ,22 2022*

At *10:10 – 11:00*

In *101-*

Misha Gavrilovich

will talk about

Convergence and homotopical triviality are defined by the same simplicial formula

Abstract: A topological structure on a set enables one to give an exact meaning to the phrase "whenever x is sufficiently near a, x has the property $P(x)$;" we introduce a notion of a generalised topological space which enables one to give a similar exact meaning to the phase "every n-tuple of sufficiently similar points $x_1, x_2, \ldots, x_n$ has property $P(x_1, \ldots, x_n)$" for $n > 1$. (Unfiorm spaces were introduced to do this for $n = 2$ and "similar" meaning "at small distance", as explained in the introduction to (Bourbaki, General Topology).)

These spaces generalise unfiorm and topological spaces, filters, and simplicial sets, and the concept is designed to be flexible enough to formulate category-theoretically a number of standard basic elementary definitions in various fields, e.g. in analysis, limit, (unfiorm) continuity and convergence, equicontinuity of sequences of functions; in algebraic topology, being locally trivial and geometric

realisation; in geometry, quasi-isomorphism; in model theory, stability, simplicity and several Shelah's dividing lines in classfiication theory.

In the talk, I shall explain that convergence and homotopical triviality are defined by the same simplicial formula using the decalage (shfit) endomorphism of the category of generalised topological spaces; that is, in some precise sense, a sequence $(a_i)_i$ converges to a point a ffi the associated map $i \mapsto a_i$ is homotopically equivalent to the constant map $i \mapsto a$, in the category of generalised topological spaces.

If time permits, I shall try to explain how a construction of geometric realisation by Amnon Besser (1998) is interpreted as defining an endomorphism of the category of generalised topological spaces. In short, the geometric realisation of a simplex is the space of upper semi-continuous maps $[0,1] \to [0 < 1 < \cdots < N]$ with Levy-Prokhorov metric. Equivalently, the set of such maps is the Hom of the simplicial sets $X.$ and $Y.$ represented by these linear orders $[0,1]$ and $[0 < 1 < \cdots < N]$. Now, we define the endofunctor to be the inner Hom $Hom(X., Y.)$ equipped with a certain extra structure depending functorially on $Y.$

The precise definition of the category of generalised topological space is simple enough to fit in the abstract: it is the category of simplicial objects of the category of filters on sets, or, equivalently, the category of finitely additive measures taking values 0 and 1 only. Thus a generalised topological space is a simplicial set equipped, for each n≤0, with a filter on the set of n-simplices such that under any face or degeneration map the preimage of a large set is large.

The exact meaning we assign to the phase "every n-tuple of (sufficiently similar) points $x_1, x_2, \ldots, x_n$ has property $P(x_1, \ldots, x_n)$" for $n > 1$, is very much the same as done in topology: In a topological space, this exact meaning of "a property $P(x)$ holds for all points sufficiently near a" is that the set $\{x : P(x)\}$ belongs to the neighbourhood filter of a point a. Similarly, in a generalised topological space, it is that the set $\{(x_1, ..., x_n) : P(x_1, ..., x_n)\}$ belongs to the "neighbourhood" filter defined on n-simplices.