

Department of Mathematics, BGU

AGNT

On Tuesday, November 8, 2022

At 12:40 – 13:40

In 201

Yotam Hendel (Université de Lille)

will talk about

On uniform number theoretic estimates for fibers of polynomial maps over finite rings of the form $\mathbb{Z}/p^k\mathbb{Z}$

Abstract: Let $f: X \rightarrow Y$ be a morphism between smooth, geometrically irreducible \mathbb{Z} -schemes of finite type. We study the number of solutions $\#\{x: f(x) = y \pmod{p^k}\}$ for prime p , positive number k , and $y \in Y(\mathbb{Z}/p^k\mathbb{Z})$, and show that the geometry and singularities of the fibers of f determine the asymptotic behavior of this quantity as p , k and y vary.

In particular, we show that $f: X \rightarrow Y$ is flat with fibers of rational singularities, a property abbreviated (FRS), if and only if $\#\{x: f(x) = y \pmod{p^k}\}/p^{k(\dim X - \dim Y)}$ is uniformly bounded in p , k and y . We then consider a natural family of singularity properties, which are variants of the (FRS) property, and provide for each member of this family a number theoretic characterization using the asymptotics of $\#\{x: f(x) = y \pmod{p^k}\}/p^{k(\dim X - \dim Y)}$.

To prove our results, we use model theoretic tools (and in particular the theory of motivic integration, in the sense of uniform p -adic integration) to effectively study the collection $\#\{x:f(x)=y \bmod p^k\}/p^{k(\dim X - \dim Y)}$. If time allows, we will discuss these methods.

Based on a joint work with Raf Cluckers and Itay Glazer.

Please Note the Unusual Place!