

Department of Mathematics, BGU

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# Operator Algebras and Operator Theory

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*On Thursday, January 3, 2019*

*At 15:00 – 16:00*

*In 101-*

Adam Dor-On (University of Illinois at Urbana-Champaign)

will talk about

## **Operator algebraic graph representation theory**

Abstract: The Toeplitz algebra of a directed graph is the  $C^*$ -algebra generated by concatenation operators on square summable sequences over finite paths of the graph. The canonical quotient by its compact operators yields the celebrated Cuntz-Krieger algebra, which is deeply connected to the subshift of finite type and automata associated with the directed graph.

When the graph has a single vertex, representations of Toeplitz algebras were studied by Davidson, Katsoulis and Pitts, originating from work of Popescu on his non-commutative disk algebra. This is accomplished by working with the WOT closed algebra generated by operators corresponding to vertices and edges in the representation. These algebras are called free semigroup algebras, and provide a non-self-adjoint perspective for studying representations of Cuntz algebras.

The classification of representations of directed graphs up to unitary equivalence was used in producing wavelet on Cantor sets by Marcolli and Paolucci

and in the study of semi-branching function systems by Bezuglyi and Jorgensen. Hence, extending the theory of free semigroup algebras to arbitrary directed graphs becomes an important endeavor, and new connections with graph theory emerge.

In this talk I will survey work I have done over the span of three years as part of a combined effort to understand representations of Toeplitz algebras of directed graphs via non-self-adjoint techniques. We will conclude with a characterization of those finite directed graphs that admit representations with *self-adjoint* WOT closed algebras generated by vertices and edges operators. This will make full circle with the theory of automata, as we will use a periodic version of the Road Coloring theorem due to Béal and Perrin, originally proved by Trahtman in the aperiodic case. This settles a question posed in a previous paper by Davidson, B. Li and myself, and is based on joint work with Christopher Linden.