Department of Mathematics, BGU

BGU Probability and Ergodic Theory (PET) seminar

On *Thursday, December ,20 2018*

At *11:00 – 12:00*

In *101-*

Ross Pinsky (Technion)

will talk about

A Natural probabilistic model on the integers and its relation to Dickman-type distributions and Buchstab's function

Abstract: Let $\{p_j\}_{j=1}^{\infty}$ denote the set of prime numbers in increasing order, let $\Omega_N \subset \mathbb{N}$ denote the set of positive integers with no prime factor larger than p_N and let P_N denote the probability measure on Ω_N which gives to each $n \in \Omega_N$ a probability proportional to $\frac{1}{n}$. This measure is in fact the distribution of the random integer $I_N \in \Omega_N$ defined by $I_N = \prod_{j=1}^{N} p_j^{X_{p_j}}$, where $\{X_{p_j}\}_{j=1}^{\infty}$ are independent random variables and X_{p_j} is distributed as $\text{Geom}(1 - \frac{1}{p_j})$. We show that $\frac{\log n}{\log N}$ under P_N converges weakly to the *Dickman distribution*. As a corollary, we recover a classical result from classical multiplicative number theory—*Mertens' formula*, which states that $\sum_{n \in \Omega_N} \frac{1}{n} \sim e^{\gamma} \log N$ as $N \to \infty$.

Let $D_{\text{nat}}(A)$ denote the natural density of $A\subset\mathbb{N}$, fi it exists, and let $D_{\text{log-indep}}(A) = \lim_{N\to\infty} P_N(A\cap\Omega_N)$ denote the density of A arising from $\{P_N\}_{N=1}^{\infty}$, fi it exists. We show that the two densities coincide on a natural algebra of subsets of $\mathbb{N}$. We also show that they do not agree on the sets of $n^{\frac{1}{s}}$- *smooth numbers* $\{n\in\mathbb{N}: p^+(n)\le n^{\frac{1}{s}}\}$, $s>1$, where $p^+(n)$ is the largest prime divisor of n. This last consideration concerns distributions involving the *Dickman function.* We also consider the sets of $n^\frac{1}{s}$-*rough numbers* ${n\in\mathbb{N}:p^-(n)\ge n^{\frac{1}{s}}}$, $s>1$, where $p^-(n)$ is the smallest prime divisor of n. We show that the probabilities of these sets, under the unfiorm distribution on $[N]={1,\ldots, N}$ and under the P_N-distribution on Ω_N, have the same asymptotic decay profile as functions of s, although their rates are necessarily dffierent. This profile involves the *Buchstab function.* We also prove a new representation for the Buchstab function.