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On the effective conductivity of the magnetized bounded partially ionized plasma with random irregularities

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Abstract. The effective conductivity σ^{eff} of a magnetized plasma with random irregularities has been studied theoretically. The main aim of this research is to construct a general theory valid to unbounded (closed Hall circuit) as well as to bounded plasma systems (opened Hall circuit) with one kind of current carrier. Our results reveal essential differences in behaviour of σ^{eff} in these cases.

The behaviour σ^{eff} following from the theoretical considerations has been confirmed in experiments on thin non-homogeneous plates of crystal p-Si in crossed electric and magnetic fields (from 0 to 15 kGs) placed in liquid He.

In the case of an open Hall circuit, σ^{eff} differs only slightly from the average $\langle\sigma\rangle$ in the whole range of the magnetic fields. In contrast, σ^{eff} may be higher than $\langle\sigma\rangle$ for a closed Hall circuit when the magnetization parameter is greater than 10.

1. Introduction

The effective transport properties of magnetized media is of the greatest interest to wide branches of laboratory plasma physics, solid-state physics and space physics. This interest has risen because any inhomogeneities can change dramatically the effective transport characteristics such as the galvanomagnetic characteristics, the thermal flux and the effective electrical conductivity.

The determination of a relation between the effective transport characteristics of a medium and its local features allows us to better define the inhomogeneity, predict the course of transport processes in disordered systems and create heterostructures with specific properties [1]. The sensitivity of the effective transport characteristics of the magnetized solid-state plasma to small random irregularities enables us, in principle, to use this feature to also define their concentration in semiconductors [2].

This problem arises also in many situations involving ionospheric and magnetospheric electrodynamics. The ionospheric conductivity is the parameter controlling the performance of the coupling of the inner and external parts of planetary atmospheres. The clarity of transmission and scattering of the extra-low-frequency electromagnetic waves through the planetary plasma shells when complicated by random clouds, is extremely important if one keeps in mind the problems of bearing, sounding and shielding. Hydromagnetic waves incident from extraterrestrial space onto the Earth is a specific example amongst many. Understanding the regularities of hydromagnetic wave transformation at the ionospheric level is a significant problem, as well as being able to do ground-based diagnostics of the magnetospheric sizes, obtaining the distribution of the cold plasma density, etc and doing electromagnetic sounding of the Earth based on the features of the waves transmitted to the Earth through, and scattered by, the ionosphere. The use of ground electromagnetic observations as a major research tool has always been treated as the key to understanding the structure and dynamics of the magnetosphere.

In practically all these fields, the question may be formulated thus: what is the integral effect of randomly non-homogeneous ionosphere, and what is the integral current caused by an external electric field?

The electrical current flowing through a non-homogeneous ionosphere produces a magnetic perturbation. The contribution of small random wind and electron concentration perturbations to the ground and magnetospheric quasi-stationary electric and magnetic fields was studied in [3, 4]. From these studies one can connect the correlation matrices of the above- and under-ionosphere magnetic fields with those of random ionospheric irregularities.

However, if our concern is with the integral magnetic field on the ground surface, that is for a distance of around 100 km from the ionosphere containing small inclusions, then the magnetic effect caused by those irregular currents is equivalent to the magnetic effect of some average current flowing along an ionosphere with an effective conductivity. By these definitions, the problem is reduced to: how to calculate the effective conductivity, even if we are provided with detailed information about either every irregularity, or the correlation properties of the random non-homogeneous field. Intuition suggests that it is necessary to substitute an average conductivity in the case of the small perturbations. In fact, it turns out [5] that in the case of magnetized media the total current will be defined by the product of the concentration perturbations by the magnetization parameter of the current carrier. Since the discovery of the strong influence of the small perturbations on the effective transport properties of the magnetized media many physicists have been intrigued by this effect (e.g. [6–9]).

This paper is organized as follows. In the rest of this section we review, briefly, existing theories of effective conductivity both for a regular media with the scalar local conductivity, and for a magnetized media, such as a solid-state semiconductor plasma or a partially ionized plasma. The local conductivity in this case has tensor characteristics. In section 2 we present an expression for the effective conductivity of a partially ionized plasma placed between two non-conductive walls. A situation of this kind arises either in laboratory experiments with open 'Hall circuit' on the semiconductor plasma (see for example [10]), or in the equatorial plasma of the E-layer bounded below by the non-conductive atmosphere and above by the weakly conductive F-layer. In section 3 we use this approach, which was first outlined and applied in [7] to the unbounded plasma systems, to evaluate the effective conductivity tensor in the bounded magnetized systems. The expressions for σ^{eff} for small perturbations of the local conductivity in strong magnetic fields are derived. Section 4 provides a discussion and summary of those results. Theoretical outcomes are compared with the results of the laboratory experiments on p-Si non-homogeneous films placed into the strong magnetic field. Finally, we discuss and consider some practical issues related to our predictions.

1.1. Existing theories

The effective conductivity σ^{eff} defines a connection between the volume average current density $\langle \boldsymbol{j} \rangle$ and electric field $\langle \boldsymbol{E} \rangle$

$$\langle \boldsymbol{j}(\boldsymbol{r}) \rangle = \sigma^{\mathrm{eff}} \langle \boldsymbol{E}(\boldsymbol{r}) \rangle \tag{1}$$

σ^{eff} is actually measured in experiments and appears in the averaged Maxwell equations. $\boldsymbol{j}(\boldsymbol{r})$ and $\boldsymbol{E}(\boldsymbol{r})$ in Ohm's law (1) are the local current and electric field. In the common case, the spatial distribution of the local conductivity $\sigma(\boldsymbol{r})$ is random.

σ^{eff} does not, in general, coincide with the average conductivity and can differ greatly from the latter. A wide range of theoretical approaches have been applied to this problem. In certain situations, as in the example of rare inclusions, it may be assumed that the medium with conductivity σ_1 contains inclusions of conductivity σ_2. Let p be the volume concentration of the inclusions. Assume also, that an external electric field is applied to the medium. Then, if $p \ll 1$ a mutual impact of inclusions may be neglected and one can consider that only the field $\langle \boldsymbol{E} \rangle$ influences the irregularities and

$$\langle \boldsymbol{j}(\boldsymbol{r}) \rangle = \sigma_1 \langle \boldsymbol{E} \rangle + p(\sigma_2 - \sigma_1)\langle \boldsymbol{E}_2 \rangle$$

where $\langle \boldsymbol{E} \rangle = (1-p)\langle \boldsymbol{E}_1 \rangle + p\langle \boldsymbol{E}_2 \rangle$ and

$$\langle \boldsymbol{E}_1 \rangle = \frac{1}{V - V_2} \int_{V-V_2} \boldsymbol{E}(\boldsymbol{r})\,\mathrm{d}v \qquad \langle \boldsymbol{E}_2 \rangle = \frac{1}{V_2} \int_{V_2} \boldsymbol{E}(\boldsymbol{r})\,\mathrm{d}v$$

where V is the total volume of the system and V_2 is the volume occupied by inclusions. $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are the fields outside and inside an irregularity, respectively. The averaging here is performed on the volume larger than the scale size of an inclusion.

On assuming that the inclusions are spherical, we have for σ^{eff} [11]

$$\sigma^{\mathrm{eff}} = \sigma_1 + p\frac{3(\sigma_2 - \sigma_1)\sigma_1}{\sigma_2 + 2\sigma_1}. \tag{2}$$

The next stage of generalization is to give up the concept of rare inclusions and to consider a medium of conductivity σ_1 in which pieces of conductivity σ_2 are imbedded. Brugemann [12] gave a method of the so-called self-consistent field. It was agreed that all non-homogeneities create a general average electric field and that an inclusion is embedded into an 'effective medium' of conductivity σ^{eff}. Every irregularity, in turn, is polarized in that field and the polarization field initiated by the irregularity is readily calculated for spherical inclusions. The general electric field induced by all inclusions is equated to the average electric field. The effective conductivity is found from the equality of these fields. For example, for binary mixtures of spherical particles of conductivities σ_1 and σ_2 and concentrations p_1 and $p_2 = 1 - p_1$, the equation for σ^{eff} has the form

$$\sigma^{\mathrm{eff}} = \frac{a + [a^2 + 8\sigma_1\sigma_2]^{1/2}}{4} \qquad \text{where } a = (3p_2 - 1)\sigma_2 + (3p_1 - 1)\sigma_1. \tag{3}$$

If one of the components is actually dielectric (for example, $\sigma_2 = 0$), σ^{eff} becomes zero. Thus, there is a critical concentration p_{c} ('percolation threshold') below which $\sigma^{\mathrm{eff}} = 0$. In the case of spherical inclusions $p_{\mathrm{c}} = 1/3$. The value of the percolation threshold is a characteristic of a wide variety of heterogenic systems. In actual composites p_{c} can vary greatly. Behaviour of the effective conductivity close to the percolation threshold is described by

$$\sigma^{\mathrm{eff}} = \sigma_1 (p - p_{\mathrm{c}})^t$$

where t is a critical index the value of which was defined by modelling of the percolation problem on different lattices [13–15].

The outlined methods for calculating σ^{eff} are based on the assumption that the mixed phases are separated by well-shaped edges. On numerous occasions, however, especially in polycrystal samples, strongly doped compensated semiconductors, non-homogeneous laboratory and cosmical plasmas and other systems with irregular spatial distribution, the conductivity σ is a continuous function of the coordinate $\boldsymbol{r}$. Herring [5] has given a theoretical treatment of the influence of random inhomogeneities on the effective electrical properties of such systems. Formulae are developed which are asymptotically exact in the limit of small fractional fluctuations of the local conductivity. A numerical approach has been proposed in which all fluctuating values are expanded as a Fourier series in spatial wavenumbers $\boldsymbol{k}$

$$\sigma(\boldsymbol{r}) = \langle\sigma\rangle + \sum_{k\neq 0} \sigma_{\boldsymbol{k}} \exp(\mathrm{i}\boldsymbol{k}\boldsymbol{r}). \tag{4}$$

Here $\sigma_{\boldsymbol{k}}$ is the amplitude of the harmonic of spatial wavenumber $\boldsymbol{k}$, and $\langle\sigma\rangle$ is the average conductivity. It has been found that in the lowest approximation for fluctuating conductivities, the effective conductivity is

$$\sigma^{\mathrm{eff}} = \langle\sigma\rangle \left[1 - \frac{\sum_k \sigma_{\boldsymbol{k}}\sigma_{-\boldsymbol{k}}}{\langle\sigma\rangle^2}\right] \tag{5}$$

and can be rewritten in the more descriptive form

$$\sigma^{\mathrm{eff}} = \langle\sigma\rangle \left[1 - \frac{\langle\delta\sigma^2(\boldsymbol{r})\rangle}{\langle\sigma\rangle^2}\right] \tag{6}$$

where $\delta\sigma(\boldsymbol{r}) = \sigma(\boldsymbol{r}) - \langle\sigma\rangle$ and $\langle\delta\sigma^2(\boldsymbol{r})\rangle$ is a mean square deviation of the local value $\sigma(\boldsymbol{r})$ from the average $\langle\sigma\rangle$.

Thus, for calculation of σ^{eff}, the spatial distribution of $\sigma(\boldsymbol{r})$ needs to be known. It is realized, that in the case of random irregularities, it would be desirable first, to determine theoretically the spectrum of irregularities and then, to calculate the resulting effective conductivity. Unfortunately, usually, we do not know the real spectrum caused by various plasma instabilities of laboratory or cosmical plasmas, and also by the impurity concentration in a semiconductor sample. However, in any case, independent of the actual form of $\sigma(\boldsymbol{r})$, the magnitude of σ^{eff} for small perturbations in accordance with (6) is always less than the mean conductivity $\langle\sigma\rangle$.

The situation changes drastically in the case of a magnetized media. Here, one of the components of the tensor of the effective conductivity $\sigma_{ij}^{\mathrm{eff}}$ becomes larger than the mean conductivity $\langle\sigma_{ij}\rangle$ even for small perturbations of the local conductivity.

In experiments with high mobility semiconductors, for example, InSb, a transverse magnetoresistance is observed to grow linearly with the magnetic field $\boldsymbol{H}_0$. However, the classical theory of transport phenomena predicts saturation of the transverse magnetoresistance in strong magnetic fields. Herring [5] was the first to notice that this phenomenon can be associated with the effect of conductivity inhomogeneities, the sizes of which are small in comparison with the sample size, but substantially exceed the mean free path of carriers. He gave a theoretical treatment of the joint effect of random inhomogeneities and magnetic field on electrical properties. The formulae developed were asymptotically exact in the limit of the small fluctuations of the local conductivity.

However in strong $\boldsymbol{H}_0$, when the magnetization parameter

$$\beta = \omega_{\mathrm{c}}\tau_n \gg 1 \tag{7}$$

the conductivity becomes anisotropic. In (7) $\omega_{\mathrm{c}} = ZeH/mc$ is the Larmor frequency, τ_n is the time of carrier relaxation, e is the elementary charge, c is the light velocity and Z and m are the charge state and mass carriers, respectively. If there is just one charge carrier, a diagonal component of the local conductivity tensor depends on H as $1/H^2$, and a non-diagonal one varies as $1/H$ ($H = |\boldsymbol{H}_0|$). The diagonal component of the effective transverse conductivity $\sigma_{\perp}^{\mathrm{eff}}(H)$ will be defined by some combination of components of local conductivity averaged over the volume. Herring in his pioneering work [5] gave an explicit expression for the correction for the transverse conductivity in the lowest order considering inhomogeneities to be small. The expansion was carried out using the value of relative fluctuation of conductivity

$$\xi = \left(\frac{\langle\delta\sigma^2\rangle}{\langle\sigma\rangle^2}\right)^{1/2}. \tag{8}$$

It was found that a correction $\delta\sigma_{\perp}^{\text{eff}}(H)$ defined by random inhomogeneities varies inversely with the magnetic field H and decreases with the growth of H slower than the non-perturbed conductivity $\sigma_{\perp}(H)$. Therefore, it was concluded that even for small perturbations ($\xi \ll 1$) in strong H, the correction may exceed the value of the undisturbed conductivity $\sigma_{\perp}(H)$. Note, that this conclusion is not formally correct because it applies perturbation theory to the case where the correction term is greater than the main term.

Dreizin and Dychne [7], Kvyatkovsky [8], Galperin and Laichtman [16] showed later that the true parameter by which the expansion should be carried out in [5] is not ξ, but $\beta\xi$. They summarized the complete series of perturbation theory and proved strictly the validity of Herring's conclusion [5]. According to [7]

$$\delta\sigma_{\perp}^{\text{eff}}(\beta) = A\left(\frac{\xi}{\beta}\right)^{\mu}\sigma_0. \tag{9}$$

Here σ_0 is the conductivity along the magnetic field and A is some constant independent of β. For a three-dimensional (3D) system the exponent is $\mu = 4/3$.

It was also indicated [8] that in the described system a specific size effect can appear—a dependence of the conductivity on the scale-size L_z along $\boldsymbol{H}_0$. Moreover, in very strong magnetic fields $\sigma_{\perp}^{\text{eff}}$ turns out to be inversely proportional to H [8]

$$\delta\sigma_{\perp}^{\text{eff}}(\beta) = \frac{\xi}{\beta}\cdot\left(\frac{a}{L_z}\right)^{1/2}\sigma_0 \tag{10}$$

where a is the inhomogeneity's size.

The method of [7, 8] was developed by Alperovich and Chaikovsky [17] and applied to the ionospheric plasma.

Coincident with the development of the theory of σ^{eff} for systems with a continuously distributed non-homogeneous conductivity, there were obtained significant theoretical and numerical results for the two-phase systems like a metal dielectric [6, 9, 18].

A special case of two-phase systems with periodical inclusions has been considered by Bergman and Strelniker [20] and Tornow *et al* [21]. They predicted an effect of the strong anisotropy of the conductivity in the strong magnetic field. The effect was confirmed in experiments with nGaAs-layers in which the periodic voids were burned through by the electronic beam [21].

Despite the fact that our understanding of anisotropic disordered systems and their effective properties has increased significantly over the past two decades, transport in bounded anisotropic disordered media is a fundamental, and major, unsolved problem. This paper is directed toward the solution of the problem of the effective conductivity of the bounded magnetized plasma system containing small-scale time-independent random irregularities of the concentration of charge carriers.

2. Homogeneous plasma

A charged particle of mass m and charge e with moving velocity $\boldsymbol{v}$ in the presence of an electric field, $\boldsymbol{E}$, and a magnetic field, $\boldsymbol{H}_0$ is subjected to two forces: an electrostatic force

$$\mathbf{F}_{\parallel} = \frac{e\boldsymbol{E}}{m} \tag{11}$$

parallel to $\boldsymbol{E}$ and the magnetic (Lorentz) force perpendicular to both $\boldsymbol{v}$ and $\boldsymbol{H}_0$

$$\boldsymbol{F}_{\perp} = \frac{e}{c}[\boldsymbol{v}\times\boldsymbol{H}_0]. \tag{12}$$

Since the conductivity σ_0 along the magnetic field is unaffected by $\boldsymbol{H}_0$, we can take $\boldsymbol{E}$, without loss of generality, to be perpendicular to $\boldsymbol{H}_0$. The plasma becomes anisotropic, i.e. the electric field $\boldsymbol{E}$ applied to the plasma produces parallel $j_\parallel$ and perpendicular $j_\perp$ to $\boldsymbol{E}$ electrical currents.

Taking the axis z along $\boldsymbol{H}_0$ and the xy-plane to contain $\boldsymbol{E}(E_x, E_y)$, the two component equations of Ohm's law are

$$j_x = \sigma_\mathrm{P} E_x + \sigma_\mathrm{H} E_y \qquad j_y = \sigma_\mathrm{P} E_y - \sigma_\mathrm{H} E_x. \tag{13}$$

The conductivities σ_P and σ_H connecting $j_\parallel$ and $j_\perp$ with the electric field are called the Pedersen and Hall conductivities, respectively.

Let us now turn to the coordinate system so that the x-axis coincides with the direction of the applied electric field $\boldsymbol{E}$. Then, we have

$$j_x = \sigma_\mathrm{P} E_x \qquad j_y = -\sigma_\mathrm{H} E_x. \tag{14}$$

In the simplest case of a homogeneous plasma system located between two long uniform non-conductive walls, the total current will not be defined by merely local σ_P and σ_H, but by their combination called the Cowling conductivity (σ_C) in cosmical physics and the Hall coefficient in solid-state physics. Assuming that the j_y-current cannot flow through the boundaries ($j_y = 0$), we have from (13)

$$E_y = \frac{\sigma_\mathrm{H}}{\sigma_\mathrm{P}} E_x \tag{15}$$

and j_x tends to

$$j_x = \sigma_{xx}^{\mathrm{eff}} E_x \qquad \text{where } \sigma_{xx}^{\mathrm{eff}} = \sigma_\mathrm{C} = \sigma_\mathrm{P} + \frac{\sigma_\mathrm{H}^2}{\sigma_\mathrm{P}}. \tag{16}$$

For the magnetized plasma with one kind of carrier the Pedersen σ_P and Hall σ_H conductivities can be written as

$$\langle\sigma_{xx}\rangle = \frac{\sigma_0}{1+\beta^2} \qquad \langle\sigma_{xy}\rangle = \frac{\sigma_0\beta}{1+\beta^2}. \tag{17}$$

Then, the relationship (16) for $\sigma_{xx}^{\mathrm{eff}}$ of the bounded magnetized 'electron' plasma reduces to

$$\sigma_{xx}^{\mathrm{eff}} = \sigma_0 = \frac{Ne^2}{m\nu_\mathrm{e}} \tag{18}$$

where σ_0 is longitudinal along the magnetic field conductivity. In this situation, the charges move along the plasma under the electric field as if there is no magnetic field. That is, the magnetoresistivity of the plasma vanishes. The effect results from the accumulation of charges of the opposite sign on the bounds of the plasma system. Their amount is such that the presented electrostatic force compensates precisely the Lorentz force.

3. Non-homogeneous plasma

3.1. Stochastic inhomogeneities and the effective conductivity

Calculation of the electrical characteristics of a weakly inhomogeneous plasma in a strong magnetic field has been carried out before in [6, 21–23]. In [22] this problem was solved exactly for the one-dimensional (1D) layer medium, in which the conductivity of each layer was a random function of the transverse coordinate. It was found that for $\beta\xi \gg 1$ (but $\xi \ll 1$) there is an essential change of its transport parameters. This result was confirmed experimentally in [23] in studies of the conductivity of turbulent partially ionized plasmas in a strong magnetic field. A strong change of the conductivity was demonstrated qualitively in

[25] for two-dimensional (2D) inclusions. An expression for the effective conductivity of a medium with randomly alternating regions of depressed and enhanced plasma density has been given in [6]. It was shown that the conductivity of such systems is inversely proportional to the applied magnetic field. We will demonstrate that inserting the boundary conditions changes radically the effective transport properties of a weakly inhomogeneous magnetized plasma.

Let us consider the stationary current flowing in the medium where the conductivity tensor $\hat{\sigma}(\boldsymbol{r})$ is a random coordinate function. Ohm's law connecting the local current density $\boldsymbol{j}(\boldsymbol{r})$ and the local electric field $\boldsymbol{E}(\boldsymbol{r})$ can be written as

$$\boldsymbol{j}(\boldsymbol{r}) = \hat{\boldsymbol{\sigma}}(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r}). \tag{19}$$

In addition $\boldsymbol{j}(\boldsymbol{r})$ and $\boldsymbol{E}(\boldsymbol{r})$ satisfy

$$\nabla \cdot \boldsymbol{j}(\boldsymbol{r}) = \mathbf{0} \qquad \nabla \times \boldsymbol{E}(\boldsymbol{r}) = \mathbf{0} \qquad \boldsymbol{E} = -\nabla\boldsymbol{\varphi} \tag{20}$$

where $\boldsymbol{\varphi}$ is a potential.

Much more interesting in practice is not (19) but rather Ohm's law for the average current $\langle j_i\rangle$ and average electric field $\langle E_k\rangle$

$$\langle j_i\rangle = \sigma_{ik}^{\mathrm{eff}}\langle E_k\rangle \tag{21}$$

where $\sigma_{ik}^{\mathrm{eff}}$ is the effective conductivity of the spatially inhomgeneous anisotropic system.

Such a statement for $\sigma_{ik}^{\mathrm{eff}}$ is valid assuming the following: (1) the free path of the current carriers should be significantly less than the scale length of inclusions; and (2) the characteristic frequency of variation of the applied electric field is small compared to the longitudinal conductivity σ_0 (along the ambient magnetic field) and to the dispersion frequency of the conductivity (is an order of collision frequency of charge carriers with neutrals). The first condition enables us to use local Ohm's law (19) and the second one to use steady Maxwell's equations (20).

Let all fluctuating values $a(\boldsymbol{r})$ be written as a sum of a mean value $\langle a\rangle$ of $a(\boldsymbol{r})$ and its fluctuating part $\delta a(\boldsymbol{r})$

$$a(\boldsymbol{r}) = \langle a\rangle + \delta a(\boldsymbol{r}). \tag{22}$$

For example, the potential φ can be written as

$$\varphi = \langle\varphi\rangle + \delta\varphi = -\langle E_k\rangle x_k + \delta\varphi \tag{23}$$

then

$$\frac{\partial\varphi}{\partial x_k} = -\langle E_k\rangle + \frac{\partial\delta\varphi}{\partial x_k}. \tag{24}$$

Taking into account that all mean values of the fluctuating parts are independent of coordinates, particularly

$$\langle\delta\varphi\rangle = 0 \qquad \langle\delta\sigma_{ik}\rangle = 0 \qquad \left\langle\frac{\partial\delta\varphi}{\partial x_k}\right\rangle = 0 \tag{25}$$

we find

$$\langle j_i\rangle = \langle\sigma_{ik}\rangle\langle E_k\rangle - \left\langle\delta\sigma_{ik}\frac{\partial\delta\varphi}{\partial x_k}\right\rangle. \tag{26}$$

Equation $\nabla \cdot \boldsymbol{j}(\boldsymbol{r}) = \mathbf{0}$ with (24) becomes

$$\frac{\partial(\delta\sigma_{ik})}{\partial x_i}\cdot\frac{\partial\delta\varphi}{\partial x_k} + \langle\sigma_{ik}\rangle\frac{\partial^2\delta\varphi}{\partial x_i\partial x_k} + \delta\sigma_{ik}\frac{\partial^2\delta\varphi}{\partial x_i\partial x_k} - \frac{\partial(\delta\sigma_{ik})}{\partial x_i}\langle E_k\rangle = 0. \tag{27}$$

Therefore, the problem becomes one of finding $\delta\varphi(\boldsymbol{r})$ with a known coordinate dependence of $\delta\sigma_{ik}(\boldsymbol{r})$. In the appendix we show a formal solution, the procedure reducing to an expansion of fluctuating variables in the Fourier series. Substitution of the series into (27) leads to a much more tractable equation for separate spatial Fourier harmonics, and, after considerable manipulation, Ohm's law for the average current is (A4)

$$\langle j_i\rangle = \langle\sigma_{ik}\rangle\langle E_k\rangle + \sum_{\boldsymbol{q}} \delta\sigma_{il}(-\boldsymbol{q})\frac{B_l(\boldsymbol{q})}{\langle\sigma_{im}\rangle q_i q_m}\langle E_l\rangle q_l \tag{28}$$

which is basic for the calculation of the effective conductivity $\sigma_{ik}^{\mathrm{eff}}$. Here $B_i(\boldsymbol{q})$ is defined by (A3). By definition, $\sigma_{xx}^{\mathrm{eff}}$ is the transverse effective conductivity for $\boldsymbol{E}\|x$ and $\boldsymbol{H}_0\|z$. Then, (A4) can be rewritten as

$$\langle j_k\rangle = \sum_{i=x,y}\left\{\langle\sigma_{ki}\rangle + \sum_{\boldsymbol{q}}\frac{\delta\sigma_{kl}(\boldsymbol{q})q_l B_i(\boldsymbol{q})}{\langle\sigma_{mn}\rangle q_m q_n}\right\}\langle E_i\rangle. \tag{29}$$

Here, $k = x, y$ for the corresponding current components. It is evident from (21) that (29) yields $\sigma_{ik}^{\mathrm{eff}}$. A variety of techniques are available for the solution of the integral equation (29). The most obvious is an iterative procedure taking into account the first approximation on the correlation function of conductivity. It means, that we must include just the first term of the integral equation to estimate $B_i(q)$ in (A3).

3.1.1. Closed Hall circuit, $\langle E_y\rangle = 0$. From (29) follows

$$\sigma_{ik}^{\mathrm{eff}} = \langle\sigma_{xx}\rangle + \delta\Sigma_{xx} \tag{30}$$

where

$$\delta\Sigma_{ij} = \sum_{\boldsymbol{q}}\frac{\delta\sigma_{il}(-\boldsymbol{q})q_l B_j(\boldsymbol{q})}{\langle\sigma_{mn}\rangle q_m q_n} \qquad i, j \equiv x, y \tag{31}$$

was analysed in detail in a series of papers (see, for example, [7, 8]) for the case of strong magnetic fields $\beta \gg 1$.

However, (31) is obtained by making very general assumptions about the magnetic field. Therefore, it is valid for both strong and weak fields. In the case of one sort of carriers, components σ_{P} and σ_{H} of the tensor conductivity are of the form (17).

Weak magnetic field, $\beta \ll 1$. For the weak magnetic field, when $\beta \ll 1$, the component $\langle\sigma_{xx}\rangle \gg \langle\sigma_{xy}\rangle$. The main contribution to (31) is provided by $\delta\sigma_{xx}(q)$. We obtain from this equation that

$$\delta\Sigma_{xx} = \sum_{\boldsymbol{q}}\frac{\delta\sigma_{xx}(-\boldsymbol{q})q_x B_x(\boldsymbol{q})}{\langle\sigma_{mn}\rangle q_m q_n}. \tag{32}$$

In the lowest approximation in fluctuations $\delta\sigma_{xx}(q)$ the quantity $B_x(\boldsymbol{q})$, which determines the perturbation of the potential caused by inclusions, (see A3) is

$$B_x(\boldsymbol{q}) \approx -\delta\sigma_{xx}(\boldsymbol{q})q_x. \tag{33}$$

Then, the addition of $\delta\Sigma_{xx}$ to the effective Pedersen conductivity $\sigma_{ik}^{\mathrm{eff}}$ in (30) can be written as

$$\delta\Sigma_{xx} \approx -\sum_{\boldsymbol{q}}\frac{\delta\sigma_{xx}(-\boldsymbol{q})\delta\sigma_{xx}(\boldsymbol{q})q_x^2}{\langle\sigma_{mn}\rangle q_m q_n}.$$

We obtain, in view of $\langle\sigma_{xx}\rangle \gg \langle\sigma_{xy}\rangle$, that

$$\delta\Sigma_{xx} \approx -\frac{\langle\delta\sigma_{xx}^2(\boldsymbol{r})\rangle}{\langle\sigma_{xx}\rangle}. \tag{34}$$

Substituting (34) into (30) we then have an expression for the effective conductivity in form (6) in which $\langle\sigma\rangle$ should be changed for $\langle\sigma_{xx}\rangle = \sigma_0/(1+\beta^2)$. It is clear, that the effective conductivity is less than the average conductivity in the case of a weak magnetized medium ($\beta < 1$).

Strong magnetic field ($\beta \gg 1$). Expressions (17) for diagonal and non-diagonal parts of the tensor conductivity reduce to

$$\sigma_{xx} \approx \frac{\sigma_0}{\beta^2} \qquad \sigma_{xy} \approx \frac{\sigma_0}{\beta}. \tag{35}$$

The main term in (31) is defined by the Hall perturbed component $\delta\sigma_{xy}$. Therefore, the correction $\delta\Sigma_{xx}$ is

$$\delta\Sigma_{xx} \approx \sum_{\boldsymbol{q}} \frac{\delta\sigma_{xy}(-\boldsymbol{q})q_y B_x(\boldsymbol{q})}{\langle\sigma_{mn}\rangle q_m q_n} \tag{36}$$

with

$$B_x(\boldsymbol{q}) \approx -\delta\sigma_{yx}(\boldsymbol{q})q_y.$$

Taking into consideration an asymmetry of $\delta\sigma_{ij}$ for $i \neq j$, that is $\delta\sigma_{xy} = -\delta\sigma_{yx}$, we obtain

$$\delta\Sigma_{xx} \approx +\sum_{\boldsymbol{q}} \frac{|\delta\sigma_{xy}(\boldsymbol{q})|^2 q_y^2}{\langle\sigma_{mn}\rangle q_m q_n}. \tag{37}$$

Hence, it follows that the contribution from inhomogeneities is positive and in the case of $\beta \gg 1$ the effective conductivity always exceeds $\langle\sigma_{xx}\rangle$.

Small perturbations ($\beta\xi \geqslant 1$). Let us estimate σ_{xx}^{eff} for small perturbations of charge carriers and for such strong magnetic fields ($\beta \gg 1$) that $\beta\xi > 1$ where ξ is the relative conductivity fluctuation defined by equation (8). Using as the starting point the equation for $\delta\Sigma_{xx}$ in the form

$$\delta\Sigma_{xx}^{(1)} = \int \frac{\langle|\delta\sigma_{xy}(\boldsymbol{q})|^2\rangle q_y^2}{\sigma_0[q_z^2 + \beta^{-2}(q_x^2 + q_y^2)]}\, \mathrm{d}\boldsymbol{q} \tag{38}$$

with fluctuations of $\delta\sigma_{xy} \sim \sigma_0\xi/\beta$. Therefore

$$\langle|\delta\sigma_{xy}(\boldsymbol{q})|^2\rangle \sim \frac{\sigma_0^2\xi^2}{\beta^2} D(\boldsymbol{q}). \tag{39}$$

Letting the correlation length of fluctuations be a, then $D(q) \approx a^3$ for wavenumbers $q \leqslant a^{-1}$ and $D(q) \approx 0$ for $q \gg a^{-1}$. Then, (38) may be written as

$$\delta\Sigma_{xx}^{(1)} = \frac{\sigma_0\xi^2}{\beta^2} \int \frac{D(\boldsymbol{q})q_y^2}{q_z^2 + \beta^{-2}(q_x^2 + q_y^2)}\, \mathrm{d}\boldsymbol{q}. \tag{40}$$

For $\beta \gg 1$ in (40) just $q_z \approx a^{-1}\beta^{-1}$ is significant and after integration by q_z in (40) we have

$$\delta\Sigma_{xx}^{(1)} = \frac{\sigma_0\xi^2}{\beta} \int D(q_\perp)\frac{q_y^2}{q_\perp}\, \mathrm{d}^2 q_\perp \tag{41}$$

where $q_\perp^2 = q_x^2 + q_y^2$. The integral in (41) is of the order of one, hence

$$\delta\Sigma_{xx}^{(1)} = \frac{\sigma_0\xi^2}{\beta}. \tag{42}$$

The series for $\delta\Sigma_{xx}$ (37) can be summarized in all exponents of the parameter $\beta\xi^2$ applying a diagram technique [7]. As a result, we obtain

$$\sigma_{xx}^{\text{eff}} \approx \sigma_0 \left(\frac{\xi}{\beta}\right)^{4/3} \tag{43}$$

and it is suggested here that

$$\delta\Sigma_{xx} > \langle\sigma_{xx}\rangle. \tag{44}$$

Equation (43) relates to the 3D case. Practically the same calculations for 2D systems lead to

$$\sigma_{xx}^{\mathrm{eff}} \approx \sigma_0 \frac{\xi}{\beta}. \tag{45}$$

Small perturbations ($\beta\xi \leqslant 1$). We find that in this case

$$\delta\Sigma_{xx} \approx \frac{\sigma_0 \xi^2}{\beta}. \tag{46}$$

3.1.2. Open Hall circuit, $\langle j_y\rangle = 0$. In this case, $\langle j_y\rangle = 0$ in (29) allows us to find $\langle E_y\rangle$ and j_x in (29) can be rewritten as

$$\langle j_x\rangle = \sigma_{xx}^{\mathrm{eff}} \langle E_x\rangle \tag{47}$$

where

$$\sigma_{xx}^{\mathrm{eff}} = \langle\sigma_{xx}\rangle + \delta\Sigma_{xx} - \frac{(\langle\sigma_{xy}\rangle + \delta\Sigma_{xy})(\langle\sigma_{yx}\rangle + \delta\Sigma_{yx})}{\langle\sigma_{xx}\rangle + \delta\Sigma_{xx}}. \tag{48}$$

When the medium is homogeneous, that is, $\delta\Sigma_{ij} = 0$ (48) reduces to (18) ($\sigma_{xx} = \sigma_0$) and the medium, as already noted, loses magnetoresistance. In contrast, to this, when the boundary is a conductor and permits flow of the transversal current ('closed Hall circuit'), the conductivity depends on the magnetic field ($\sigma_{xx} = \sigma_0/\beta^2$). This suggests that in the case $\langle j_y\rangle = 0$ and with weak deviations from homogeneity, the fluctuating addition $\delta\sigma_{xx}^{\mathrm{eff}}$ is to be compared with $\langle\sigma_0\rangle$ but not $\langle\sigma_0/\beta^2\rangle$, and with increasing magnetic field it should not exceed $\langle\sigma_0\rangle$ (see (6)). Most probably, it will not lead to the effect of anomalous effective conductivity.

It is easy to show, that in this approximation

$$\delta\Sigma_{xy}^{(1)} = \delta\Sigma_{yx}^{(1)}. \tag{49}$$

Then, we have from (48)

$$\frac{\sigma_{xx}^{\mathrm{eff}}}{\langle\sigma_{xx}\rangle} = 1 + \frac{\delta\Sigma_{xx}}{\langle\sigma_{xx}\rangle} + \frac{\langle\sigma_{xy}\rangle^2}{\langle\sigma_{xx}\rangle(1 + \delta\Sigma_{xx}/\langle\sigma_{xx}\rangle)}. \tag{50}$$

Hence, one can obtain the inequality

$$\sigma_{xx}^{\mathrm{eff}} \geqslant 2\langle\sigma_{xy}\rangle \tag{51}$$

Small perturbations $\xi \ll 1$

(a) $\beta\xi^2 \gg 1$. In the approximation of the first iteration $\delta\Sigma_{xx} = \delta\Sigma_{xx}^{(1)}$ and for strong fields so that from (50) we obtain

$$\sigma_{xx}^{\mathrm{eff}} \approx \delta\Sigma_{xx}^{(1)} + \frac{\langle\sigma_{xy}\rangle^2}{\delta\Sigma_{xx}^{(1)}}. \tag{52}$$

Substituting (42) into (52), we have

$$\sigma_{xx}^{\mathrm{eff}} \approx \frac{\sigma_0}{\beta}\left(\xi^2 + \frac{1}{\xi}\right) \approx \frac{\sigma_0}{\beta\xi} \tag{53}$$

or

$$2\frac{\sigma_0}{\beta} < \sigma_{xx}^{\text{eff}} < \sigma_0.$$

Taking into account (52) and (44) we obtain

$$\sigma_{xx}^{\text{eff}} \approx \frac{\sigma_0}{(\beta\xi^2)^{2/3}}. \tag{54}$$

However, contrast, to the homogeneous case, σ_{xx}^{eff} depends on the magnetic field in a weakly perturbed plasma. This statement is valid even when the relaxation time is independent of energy.

(b) $\beta\xi^2 \leqslant 1$. On simple algebraic rearrangement σ_{xx}^{eff} becomes

$$\sigma_{xx}^{\text{eff}} \approx \sigma_0(1 - \beta\xi^2). \tag{55}$$

We see that σ_{xx}^{eff} decreases with increasing magnetic field (β) until $\beta\xi^2 \sim 1$ and is less than σ_{xx}^{eff} in the homogeneous bounded plasma.

The following is a summary of theoretical considerations.

4. Discussion and summary

4.1. A simple example. Isolated 2D inclusion

In previous sections we dealt with continuous models in conductivity. One can illustrate the unusual behaviour of effective conductivity in the strong magnetic field on the simplest example of an isolated cylindrical inclusion with tensor conductivity $\hat{\sigma}_2$ embedded in a medium of $\hat{\sigma}_1$ conductivity. To keep the algebra as simple as possible, we suppose that the external magnetic field is applied along the cylindrical axis and that the electric field is perpendicular to the cylinder. This example is identical to a circle inhomogeneity with a tensor conductivity $\hat{\sigma}_2$ placed on a thin conductive sheet with conductivity $\hat{\sigma}_1$ (see figure 1). A magnetic field $\boldsymbol{H}_0$ is perpendicular to the sheet plane. We choose the cylindrical coordinate system (r, φ) with origin in the circle centre also letting an electric field $\boldsymbol{E}$ be applied within the sheet. This means, that a homogeneous current $\boldsymbol{J}_0$ is created far from the irregularity. The inhomogeneity leads to the appearance of an anomaly current $\boldsymbol{i}$ with components j_r and j_φ.

The boundary conditions are the normal component of the current, j_r and the tangential component of the electric field, E_φ, continuous at $r = a$. Then the desired solution of the Laplace equation for the j_r and j_φ-components of the total current outside the irregularity is

$$\begin{pmatrix} j_r \\ j_\varphi \end{pmatrix} = I_0\left[1 + \frac{a^2}{r^2}\begin{pmatrix} A & -B \\ B & A \end{pmatrix}\right]\begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} \tag{56}$$

where

$$A = \frac{\rho_1^2 - \rho_2^2 - (\delta_1 - \delta_2)^2}{(\rho_1 + \rho_2)^2 + (\delta_1 - \delta_2)^2} \qquad B = \frac{2\rho_1(\delta_1 - \delta_2)}{(\rho_1 + \rho_2)^2 + (\delta_1 - \delta_2)^2}$$

$$I_0 = E\sqrt{\sigma_{\text{P1}}^2 + \sigma_{\text{H1}}^2} \qquad \delta_{1,2} = \frac{\sigma_{\text{H}1,2}}{\sigma_{\text{H}1,2}^2 + \sigma_{\text{P}1,2}^2} \qquad \rho_{1,2} = \frac{\sigma_{\text{P}1,2}}{\sigma_{\text{H}1,2}^2 + \sigma_{\text{P}1,2}^2}.$$

For σ_{P} and σ_{H} defined by (17)

$$\rho_{1,2} = \frac{1}{\sigma_{01,2}} \qquad \delta_{1,2} = \frac{\beta}{\sigma_{01,2}} \tag{57}$$

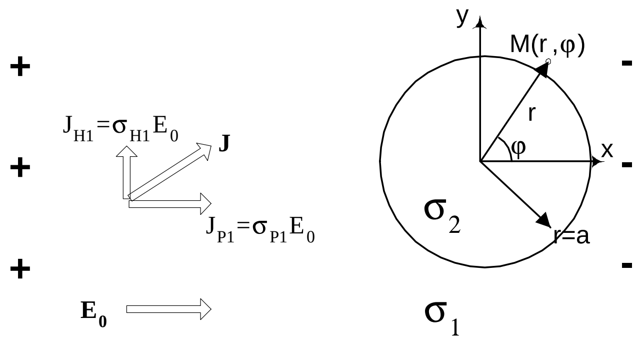

Figure 1. A sketch of a thin layer of a tensor conductivity $\hat{\sigma}_1$ with diagonal component σ_{P1} (Pedersen conductivity) and non-diagonal component σ_{H1} (Hall conductivity) and an isolated circle irregularity of a conductivity $\hat{\sigma}_2$ placed into the cross electric E_0 and magnetic $\boldsymbol{H}_0$ fields. $\boldsymbol{J}_{P1}$, $\boldsymbol{J}_{H1}$ and $\boldsymbol{J}$ are, respectively, the Pedersen, Hall and total currents far from the irregularity. The magnetic field faces into the page.

where σ_0, with corresponding index, is the longitudinal along the magnetic field conductivity outside the inclusion (index '1') or the irregularity itself (index '2') (see (18)).

Let the origin of the Cartesian coordinate system (x, y) be in the centre of the circle irregularity and the x-axis be along the initial electric field $\boldsymbol{E}(E_x, 0)$. Then the total current along the x-direction can be written as

$$j_x = j_{P1} + \frac{a^2}{r^2}\frac{j_{P1}(\rho_1^2 - \rho_2^2) + 2j_{H1}\rho_1(\delta_1 - \delta_2)}{(\rho_1 + \rho_2)^2 + (\delta_1 - \delta_2)^2} \tag{58}$$

where $j_{P1} = \sigma_{P1}E_x$ is the Pedersen current far from the irregularity. The second term in (58) is the component defined by the irregularity and consists of two parts. The first, proportional to $(\rho_1 - \rho_2)$, is the anomaly Pedersen current and the second is the term defined by the anomaly of the Hall conductivity. Let us suppose that the conductivity of the irregularity σ_{02} differs weakly from the background conductivity σ_{01} and

$$\varepsilon = \frac{\sigma_{02} - \sigma_{01}}{\sigma_{01}} \qquad |\varepsilon| \ll 1. \tag{59}$$

Then, total j_x depending on $\gamma = |\beta\varepsilon|$ with $\beta \gg 1$ is given by

$$j_x \simeq E_x\sigma_{p1}\begin{cases}\left(1 + 2\dfrac{a^2}{r^2}\dfrac{\beta}{\gamma}\right) & \gamma^2 \gg 1\\ \left(1 + \dfrac{a^2}{2r^2}\beta\right) & \gamma^2 \approx 1\\ \left(1 + \dfrac{a^2}{2r^2}\beta\gamma\right) & \gamma^2 \ll 1.\end{cases} \tag{60}$$

The quantity in braces containing γ gives us the correction caused by the irregularity to the initial j_x-current. The important point to note is that the small perturbation of the conductivity significantly affects the Pedersen current. For example, if $\gamma \gg 1$ the anomaly current exceeds the initial current by $1/\varepsilon$ times the modest distances ($r \sim a$) from the irregularity.

Suppose now, that there are two non-conductive plane boundaries parallel to the x-axis on both sides far from the irregularity. We direct an initial electric field $\boldsymbol{E}$ along x then the transversal Hall current along the y-axis vanishes ($j_{H1} = 0$). Then, from (18) it follows that

$\sigma_{xx} = \sigma_{\mathrm{P}} = \sigma_0$ and (58) tends to

$$j_x \simeq \sigma_{01} E_x \left(1 + 2\frac{a^2}{r^2}\varepsilon\right). \tag{61}$$

Hence, the contribution from the irregularity in this case is small $0(\varepsilon)$ compared to the perturbation of local conductivity.

The basic idea is adequately illustrated by the present elementary example and shows unlimited growth of the effective Pedersen conductivity for small local irregularities with increasing β in unbounded systems (see (60)) and, *vice versa*, a very slight sensitivity of the bound systems to such perturbations.

4.2. Effective 2D medium

Returning to σ^{eff}, we use the method of 'effective medium' [12, 24] to treat a model of a thin conductive layer containing a binary mixture of circle irregularities with conductivities $\hat{\sigma}_1$ and $\hat{\sigma}_2$. The second term on the right-hand side of (56) contains the r- and φ-components of a dipole moment of the isolated circle irregularity placed into crossed electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}_0$ fields. It is not difficult to write the total dipole moment of the ensemble of such irregularities. Let us suppose, that there are N_1 inclusions of radius a and conductivity $\hat{\sigma}_1$, and N_2 inclusions of radius b and conductivity $\hat{\sigma}_2$. Assuming that the whole region is occupied by non-intersecting inclusions and denoting $x_1 = N_1 a^2$, $x_2 = N_2 b^2$, then x_1 and x_2 must satisfy

$$x_1 + x_2 = 1.$$

The total polarization caused by all the irregularities should be zero. After some algebraic manipulations with the expressions in brackets of (56), we can write the system of equations to find $\sigma_{\mathrm{P}}^{\mathrm{eff}}$ and $\sigma_{\mathrm{H}}^{\mathrm{eff}}$

$$\sum_{i=1,2} x_j \frac{\sigma_{\mathrm{P}i}^2 - (\sigma_{\mathrm{P}}^{\mathrm{eff}})^2 + (\sigma_{\mathrm{H}}^{\mathrm{eff}} - \sigma_{\mathrm{H}i})^2}{(\sigma_{\mathrm{P}}^{\mathrm{eff}} + \sigma_{\mathrm{P}i})^2 + (\sigma_{\mathrm{H}}^{\mathrm{eff}} - \sigma_{\mathrm{H}i})^2} = 0 \tag{62}$$

$$\sum_{i=1,2} x_j \frac{\sigma_{\mathrm{H}i} - \sigma_{\mathrm{H}}^{\mathrm{eff}}}{(\sigma_{\mathrm{P}}^{\mathrm{eff}} + \sigma_{\mathrm{P}i})^2 + (\sigma_{\mathrm{H}}^{\mathrm{eff}} - \sigma_{\mathrm{H}i})^2} = 0 \tag{63}$$

where $j = 2, 1$ respectively for $i = 1, 2$.

Figure 2 shows the dependence of the ratio of the effective Pedersen conductivity $\sigma_{\mathrm{P}}^{\mathrm{eff}}$ to the average Pedersen conductivity $\langle\sigma_{\mathrm{P}}\rangle$ on the magnetization parameter $\beta = \omega_{\mathrm{H}}\tau_{\mathrm{c}}$. σ_{01} and σ_{02} refer to the local conductivities of the inclusions of the first and second kind, respectively. One of the curves with $x_1 = 0.5$ relates to the case when the areas of the two phases are equal. The curve with $x_1 = 0.1$ shows $\sigma_{\mathrm{P}}^{\mathrm{eff}}(\beta)/\langle\sigma_{\mathrm{P}}(\beta)\rangle$ in which 10% of the whole area is occupied by the high conductive component of $\sigma_{01} = 1$ whereas the rest of the mixture is the phase with $\sigma_{02} = 0.9$. One can see, that the effective conductivity exceeds the average conductivity by a factor of five even when the medium is weakly perturbed. The ratio $\sigma_{\mathrm{P}}^{\mathrm{eff}}(\beta)/\langle\sigma_{\mathrm{P}}(\beta)\rangle$ can be estimated crudely from

$$\frac{\sigma_{\mathrm{P}}^{\mathrm{eff}}(\beta)}{\langle\sigma_{\mathrm{P}}(\beta)\rangle} \sim \beta\xi \tag{64}$$

with ξ defined by (8).

In a manner similar to the way we obtain (61) for an isolated inclusion, we can construct 'an effective bounded medium' in the strong magnetic field. By virtue of the fact that the

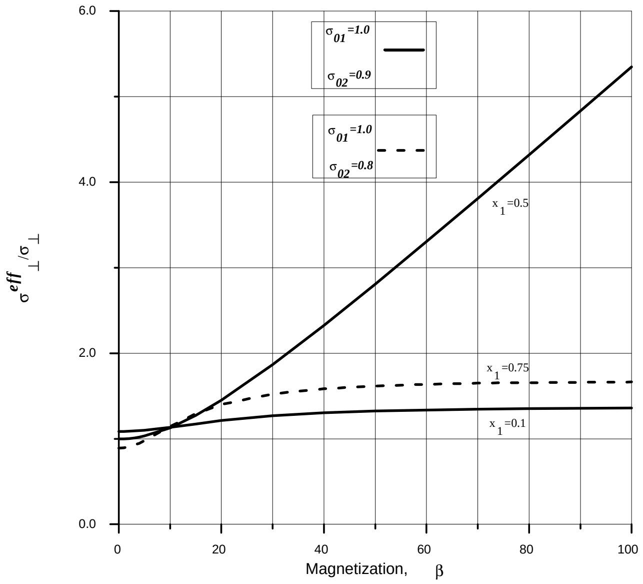

Figure 2. A plot of the ratio of the effective Pedersen conductivity $\sigma_{\rm P}^{\rm eff}$ to the spatial average Pedersen conductivity $\langle\sigma_{\rm P}\rangle$ as a function of the magnetization parameter $\beta = \omega_{\rm H}\tau_{\rm c}$. $\hat{\sigma}_{01}$ and $\hat{\sigma}_{02}$ refer to the local conductivities of the inclusions of the first and second kind, respectively. The curve with $x_1 = 0.5$ relates to the case when areas of two phases are equal. The curve with $x_1 = 0.1$ shows $\sigma_{\rm P}^{\rm eff}(\beta)/\langle\sigma_{\rm P}(\beta)\rangle$ in which 10% of the whole area is occupied by high conductive components of $\sigma_{01} = 1.0$, whereas the rest of the mixture is the phase of $\sigma_{01} = 0.9$.

dependence on the magnetic field drops out in this case, for $\sigma^{\rm eff}$ we can use (62) putting $\sigma_{{\rm H}i} = \sigma_{\rm H}^{\rm eff} = 0$. Equation (62) becomes

$$\sigma^{\rm eff} = (\sigma_{02} - \sigma_{01})(x_1 - \tfrac{1}{2}) + \tfrac{1}{2}((\sigma_{01} + \sigma_{02})^2 - 4x_1x_2(\sigma_{01} - \sigma_{02})^2)^{1/2}. \qquad (65)$$

Hence, for example, for a mixture with equal portions of two phases $x_1 = x_2 = 0.5$ we have $\sigma^{\rm eff} = (\sigma_{01}\sigma_{02})^{1/2}$. If $\sigma_{01} = 1$ and $\sigma_{02} = 0.9$, as in figure 1, then

$$\frac{\sigma^{\rm eff}}{\langle\sigma_{\rm P}\rangle} = 2\frac{(\sigma_{01}\sigma_{02})^{1/2}}{\sigma_{01} + \sigma_{02}} \sim 0.6$$

independent of the intensity of the applied magnetic field as distinct from the open system in which $\sigma^{\rm eff}/\langle\sigma_{\rm P}\rangle \sim 2$–5 for $\beta \sim 30$–100.

4.3. Experimental laboratory corroboration

The type of behaviour predicted in the preceding theoretical discussion has been confirmed in experiments with semiconductor films in strong magnetic fields [27]. Essentially, the idea was to create, in a homogeneous semiconductor, a stochastic distribution of current carriers $n(r)$ by an illumination through a special masking film in which different sectors have different transparency. Then $n(r)$ in the plate is in inverse relation to the local transparency in the mask and the conductivity becomes coordinate dependent.

Plates of silicon Si with a hole conductivity (p-Si) placed in liquid He were chosen as the study object. At liquid helium temperatures with lack of illumination the concentration of free carriers is insignificant. Then under illumination, the number of carriers rises and they determine totally the electrical current. Using different masks it was possible to change the

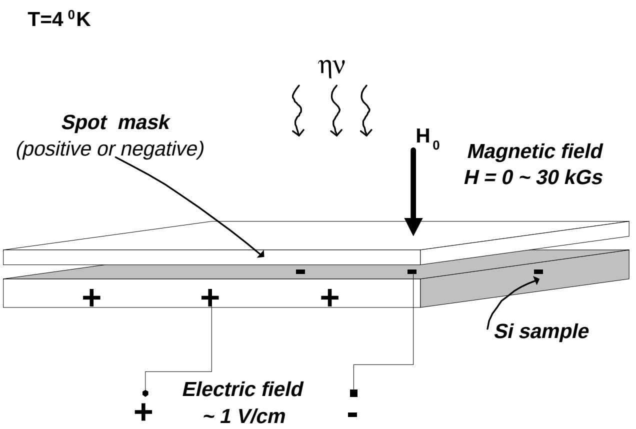

Figure 3. Schematic drawing of the modelling experiment to clarify an influence of random inclusions on the effective conductivity of anisotropic media. Plates of crystal Si at the liquid helium temperature were placed into crossed electric ($E \approx 1$ V cm^{-1}) and magnetic (0–30 kGs) fields. The samples were illuminated through either positive (the background transparency less than the spot transparency) or negative (transparency in window (spot) is less than the background one) spot masks.

level of inhomogeneity, the sizes, forms and distances between inhomogeneities in a sample. A schematic drawing of the experiment is shown in figure 3.

The measurements were taken on samples which are thin (5×10^{-2} cm) either rectangular (figure 4(a)) or disk-shaped plates with an aperture in the centre (figure 4(c)). The outer diameter of the plate was 1.0 cm and the inner 0.4 cm. A potential difference was applied to side surfaces of the rectangular plate (figure 4(a)) and large and small radii of the disk (figure 4(c)). The magnetic field was perpendicular to the plane of the plate. Free carriers were generated by a background radiation passing through a filter of pure Si and a thin sapphire mask of 0.02 cm thick pressed to the sample. One of the masks was chosen of size 0.4×0.4 cm^2, and there were 100 transparent spots sized $2L_{\mathrm{p}} \sim 0.02$ cm. The mean distance between the spot centres was $2L_{\mathrm{c}} \sim 0.4$ cm (see figure 4(d)). The level of inhomogeneity, that is the ratio of transparency in a spot to transparency outside the spot was 0.8.

The experiments were performed with samples of Si with boron impurity of concentration $N_{\mathrm{B}} \sim 6 \times 10^{15}$ cm^{-3}. The lifetime (τ) of the photo-excited carriers was governed by the impurity of the second kind of concentration N_k which was selected so that concentration inhomogeneities would be smeared by diffusion to the least possible degree. The value of τ is connected with N_k and the capture cross section s as $\tau = 1/(sN_k\langle v_{\mathrm{T}}\rangle)$, where $\langle v_{\mathrm{T}}\rangle$ is the mean thermal velocity. In the experiments N_k were; (1) $N_k \sim 5 \times 10^{13}$ cm^{-3}, (2) $N_k \sim 10^{13}$ cm^{-3}, (3) $N_k \sim 2 \times 10^{12}$ cm^{-3}. Hence, for example, for $N_k \sim 5 \times 10^{13}$ cm^{-3}, $T = 4.2$ K and typical $s \approx 10^{-14}$ cm^2 the lifetime $\tau \approx 3 \times 10^{-8}$ s. An additional condition imposed on the $\boldsymbol{E}$ value was that the drift length $L_{\mathrm{dr}} \sim \mu E \tau$ (μ is the carrier mobility) should be small in comparison with the distance between inhomogeneities, L_{c}. At low-temperature and weak electric field E, the mobility μ is defined by electron–impurity collisions. For the selected sample $\mu = 5 \times 10^4$ cm^2 V s^{-1} and $\tau \approx 3 \times 10^{-8}$ s the condition $L_{\mathrm{dr}} \ll L_{\mathrm{c}}$ holds for $E \approx 1$ V cm^{-1}. The magnetic field was changed in the range $0 < H < 30$ kOe, which corresponded to $0 < \beta < 15$.

Figure 5 shows the experimental ratio $\sigma_{xx}^{\mathrm{eff}}/\langle\sigma_{xx}\rangle$ of the sample with $N_k = 5 \times 10^{13}$ cm^{-3} as a function of the magnetization parameter, β, for the closed Hall circuit (Korbino's disk).

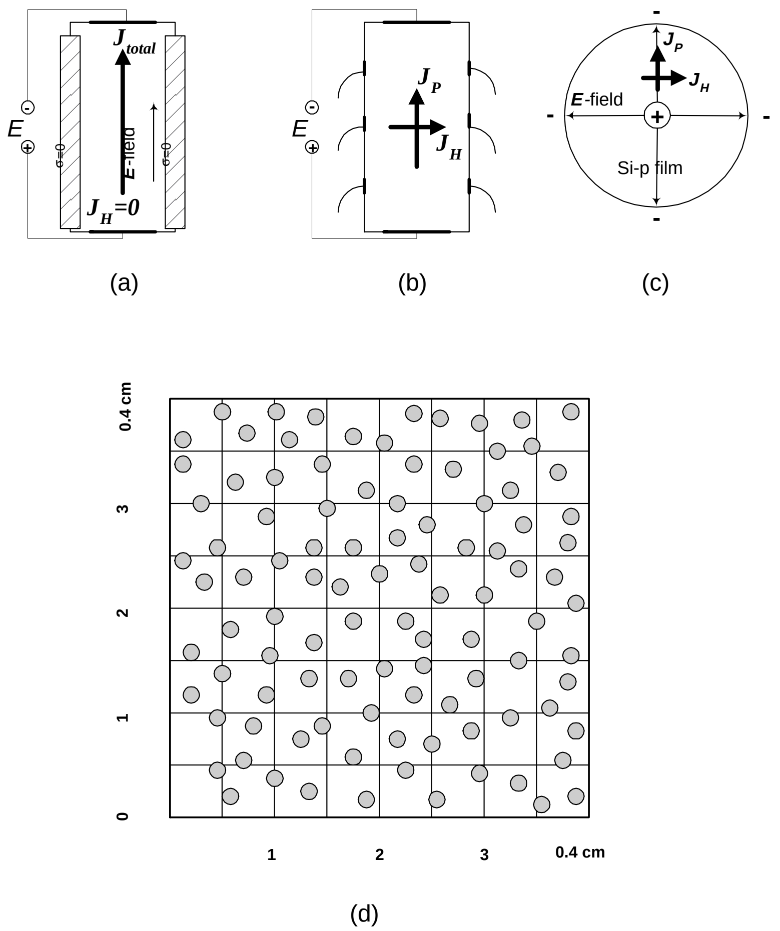

Figure 4. Geometry of the applied electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields, as well as the Pedersen current $\boldsymbol{J}_{\mathrm{P}}$ (along the electric field), Hall current $\boldsymbol{J}_{\mathrm{H}}$ (across $\boldsymbol{E}$ and $\boldsymbol{H}$) and a total current $\boldsymbol{J}_{\mathrm{T}}$. The magnetic field faces into the page. (a) Open Hall circuit. Non-conductive walls prevent $\boldsymbol{J}_{\mathrm{H}}$ flow. The vertical thin arrow indicates the applied electrical field, while the bold arrow signifies the total current as a consequence of the combined Pedersen current produced by the initial electric field and Hall current due to the polarized electric field between the walls as well. A potential difference was applied to side surfaces of the rectangular plate. (b) Closed Hall circuit in which $\boldsymbol{J}_{\mathrm{H}}$ can leak independently on $\boldsymbol{J}_{\mathrm{P}}$. (c) The measurements on the closed Hall circuit were taken on a thin (0.05 cm) disk-shaped plate with an aperture at the centre. An electric field between large and small radii of the disk exites a circular ring of the Hall current enclosed and circulating around the axis of the disk. (d) Enlarged fragment of the mask (0.4 × 0.4 cm). Different areas of the mask have different transparency.

The applied electric field $E = 0.8$ V cm^{-1} in the case of the negative mask (dark spots on the transparent background) and $E = 2.0$ V cm^{-1} for the positive mask (the background transparency less than the spot transparency). The influence of $\boldsymbol{H}$ on the conductivity becomes visible only when $\beta > 1$. In particular, at $\beta = 15$ the value of $\sigma_{xx}^{\mathrm{eff}}/\langle\sigma_{xx}\rangle$ reaches 2.1.

It was established also, that in the case of the open Hall circuit, the effective conductivity differs only weakly from the average of the whole range of magnetic fields.

For the qualitative comparison of the theoretical predictions and experimental results let us estimate the value of ξ from the relation for a 2D film. Equations (17) and (45) yield, for

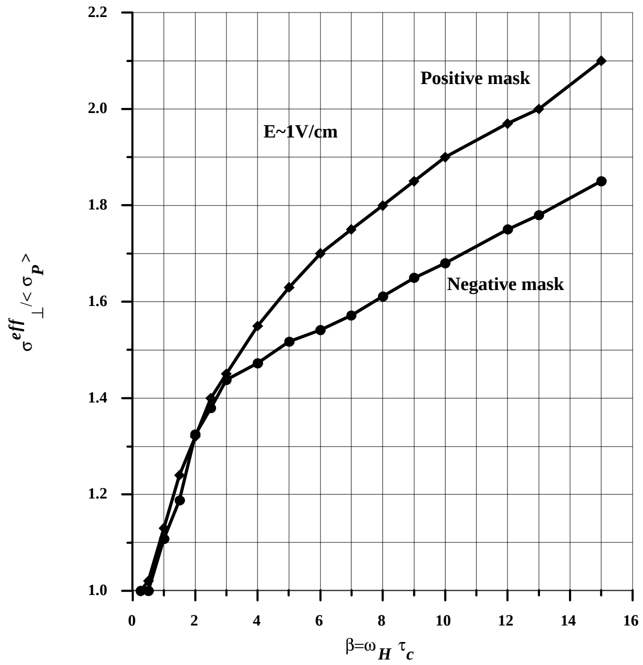

Figure 5. Ratio of the effective $\sigma_{\perp}^{\text{eff}}$ to the spatial average Pedersen conductivity $\langle\sigma_{\text{P}}\rangle$ against the magnetization parameter β for a measurement Si-sample with boron impurity of concentration $N = 6 \times 10^{15}$ cm^{-3} and the impurity of the second kind of $N_k = 5 \times 10^{13}$ cm^{-3}. The ratio of the transparencies was 0.8. Two curves represent $\sigma_{\perp}^{\text{eff}}/\langle\sigma_{\text{P}}\rangle$ for positive and negative masks.

the strong magnetic field ($\beta \gg 1$)

$$\xi = \frac{\sigma^{\text{eff}}}{\sigma_{xx}^{0}} \times \frac{1}{\beta} = \frac{2.1}{15} \approx 0.14. \tag{66}$$

At the same time, we can estimate ξ directly from the definition (8) for the 2D two-phase system: the background (phase 1) and the spots (phase 2). In this case, ξ depends on the ratio transparencies of the spots and the background and the ratio of spot areas to the total area as well. For the system consisting of randomly located regions with conductivities σ_{01} and σ_{02} and occupying area portions proportional to x_1 and x_2, respectively, the average conductivity $\langle\sigma_0(r)\rangle$ is

$$\langle\sigma_0(r)\rangle = \int \sigma_0(r)\, f(\sigma_0)\, \mathrm{d}\sigma_0 = x_1\sigma_{01} + x_2\sigma_{02} \tag{67}$$

with the distribution function $f(\sigma_0)$ defined as

$$f(\sigma_0) = x_1\delta(\sigma_0 - \sigma_{01}) + x_2\delta(\sigma_0 - \sigma_{02}) \qquad \text{with } x_1 + x_2 = 1$$

and

$$\langle\delta\sigma_0^2\rangle = \langle\sigma_0^2(r)\rangle - \langle\sigma_0\rangle^2 = x_1x_2(\sigma_{01} - \sigma_{02})^2. \tag{68}$$

Rewriting (8) in terms of (67) and (68) we obtain

$$\xi = \sqrt{\frac{x_2}{x_1}} \times \frac{|1 - \sigma_{02}/\sigma_{01}|}{1 + (x_2/x_1)(\sigma_{02}/\sigma_{01})}. \tag{69}$$

In this experiment when we deal with the negative mask, index 1 refers to areas under transparent sectors of the mask, index 2 refers to the spot areas under semitransparent sectors. Then $\sigma_{02}/\sigma_{01} \approx 0.8$ and $x_2/x_1 \approx 1/3$, using (69), yields $\xi \approx 0.1$.

One can see, that the application of the developed theory of the effective conductivity, for example, to the definition of the correlation function ξ of a disordered system in the strong magnetic fields is confirmed by the exact values of ξ defined by the geometrical and optical parameters of the mask.

5. Conclusions

1. A general expression was found for the effective conductivity σ^{eff} valid for both bounded and unbounded magnetized disordered plasma systems. From the obtained relations the essentially distinct dependence of σ^{eff} from the applied magnetic field for the open and closed systems follows.
   - (a) σ^{eff} of the unbounded systems is extremely sensitive to the small local perturbations of the charge concentration.
   - (b) Non-conductive walls bounding a system remove this feature. The σ^{eff} of systems does not differ practically from the mean conductivity. Increasing the magnetic fields in such configurations has a weak influence on the value of σ^{eff}. The range of change in σ^{eff} was found to be less than longitudinal along the magnetic field conductivity σ_0 but more than the ratio $2\sigma_0/\beta$.
2. The results may be useful for interpretation of the simultaneous observations of small- and large-scale current perturbations in the ionosphere. One can assert, for example, that σ^{eff} of the equatorial electrojet region is almost non-sensitive to sporadic electron irregularities. Conversely, σ^{eff} of the middle latitude ionosphere and the ionosphere of the outlying regions of the polar electrojet and the polar cap can be changed drastically even during small perturbations of the electron density. Formally, our reasoning is valid only for the narrow region of the lower ionosphere where both the Pedersen and Hall conductivities are defined totally by electrons.
3. We conclude that the control of the semiconductor purity based on the anomaly sensitivity of the magnetized semiconductor plasma to random carrier irregularities [2] should be performed on the closed Hall circuits like the Corbino disk rather than the usual plates.
4. We assumed in this paper that the magnetization parameter β is constant. However, such an assumption is valid only if the collision frequency ν for scattering of charged particles of species by neutrals (or crystal lattice) is independent of the energy of the colliding particles. The general theory involving $\beta \neq$ constant should lead to new, non-destructive techniques for the control of purity of semiconductors and deeper insights into their transport properties.

The contention regarding zero magnetoresistance of the bounded plasma with one kind of charge carrier ceases to be true in the situation when the role of other carriers becomes noticeable, for example, in the ionospheric plasma above around 100 km where the Pedersen conductivity is defined by ions, whereas the Hall conductivity is by electrons. A similar situation takes place in the semiconductor plasma in which a mobility of electrons and holes or heavy and light holes have similar values. In view of the major role played by small irregularities in the behaviour of σ^{eff} presently found in the plasma systems with one sort of carrier, the mixture of different carriers merits further study.

Appendix

Let us expand the fluctuating variables in the Fourier series in spatial harmonics with wavenumbers $\boldsymbol{q}$

$$\begin{pmatrix} \delta\sigma_{ik} \\ \partial(\delta\sigma_{ik})/\partial x_i \\ \partial^2\delta\varphi/\partial x_i\partial x_k \\ \partial(\delta\sigma_{ik})/\partial x_i \\ \partial\delta\varphi/\partial x_k \end{pmatrix} = \sum_{\boldsymbol{q}} \begin{pmatrix} \delta\sigma_{ik}(\boldsymbol{q}) \\ \mathrm{i}q_i\delta\sigma_{ik}(\boldsymbol{q}) \\ -q_iq_k\delta\varphi(\boldsymbol{q}) \\ \mathrm{i}q_i\delta\sigma_{ik}(\boldsymbol{q}) \\ \mathrm{i}q_k\delta\varphi(\boldsymbol{q}) \end{pmatrix} \exp\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}). \tag{A1}$$

Substituting these relations into (27) we get

$$\begin{aligned} &\sum_{\boldsymbol{q}} \mathrm{i}q_i\delta\sigma_{ik}(\boldsymbol{q})\exp\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r})\langle E_k\rangle + \langle\sigma_{ik}\rangle\sum_{\boldsymbol{q}} q_iq_k\delta\varphi(\boldsymbol{q})\exp\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}) \\ &\qquad + \sum_{\boldsymbol{q}}\sum_{\boldsymbol{p}} q_ip_k\delta\sigma_{ik}(q)\delta\varphi(\boldsymbol{p})\exp\mathrm{i}((\boldsymbol{q}+\boldsymbol{p})\cdot\boldsymbol{r}) \\ &\qquad + \delta\sigma_{ik}\sum_{\boldsymbol{q}} q_iq_k\delta\varphi(\boldsymbol{q})\exp\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}) = 0 \end{aligned}$$

which can be rewritten as

$$\begin{aligned} &\mathrm{i}q_i\delta\sigma_{ik}\langle E_k\rangle + q_iq_k\delta\varphi(\boldsymbol{q})\langle\sigma_{ik}\rangle + \sum_{\boldsymbol{p}}(q_i - p_i)p_k\delta\sigma_{ik}(\boldsymbol{q}-\boldsymbol{p})\delta\varphi(\boldsymbol{p}) \\ &\qquad + q_iq_k\delta\sigma_{ik}(\boldsymbol{q}-\boldsymbol{p})\delta\varphi(\boldsymbol{q}) = 0. \end{aligned}$$

We deduce from this that

$$\delta\varphi(\boldsymbol{q}) = \mathrm{i}\frac{B_k(\boldsymbol{q})}{\langle\sigma_{im}\rangle q_iq_m}\langle E_k\rangle \tag{A2}$$

where $B_k(\boldsymbol{q})$ satisfies

$$B_k(\boldsymbol{q}) = -\delta\sigma_{ik}(\boldsymbol{q})q_i - \sum_{\boldsymbol{q}'\neq 0}\frac{q_i\delta\sigma_{il}(\boldsymbol{q}-\boldsymbol{q}')q_l'}{\langle\sigma_{mn}\rangle q_m'q_n'}B_k(\boldsymbol{q}'). \tag{A3}$$

If we substitute for $\delta\sigma_{ik}$ and $\partial\delta\varphi/\partial\delta x_k$ from (A1), then the relationship (26) between the total current and perturbations of conductivity and potential becomes

$$\begin{aligned} \langle j_i\rangle &= \langle\sigma_{ik}\rangle\langle E_k\rangle - \left\langle\sum_{\boldsymbol{q}}\delta\sigma_{ik}(\boldsymbol{q})\exp\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r})\sum_{\boldsymbol{p}}\mathrm{i}p_k\delta\varphi(\boldsymbol{p})\exp\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{r})\right\rangle \\ &= \langle\sigma_{ik}\rangle\langle E_k\rangle - \mathrm{i}\sum_{\boldsymbol{q}}\delta\sigma_{ik}(-\boldsymbol{q})\delta\varphi(\boldsymbol{q})q_k. \end{aligned}$$

Because, by definition

$$\langle f(\boldsymbol{r})\rangle = \frac{1}{V}\int f(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}$$

then

$$\begin{aligned} &\langle\exp\mathrm{i}(\boldsymbol{q}+\boldsymbol{p})\cdot\boldsymbol{r}\rangle = 1 \qquad \text{if } \boldsymbol{q}+\boldsymbol{p} = 0 \\ &\langle\exp\mathrm{i}(\boldsymbol{q}+\boldsymbol{p})\cdot\boldsymbol{r}\rangle = 0 \qquad \text{if } \boldsymbol{q}+\boldsymbol{p} \neq 0. \end{aligned}$$

Substituting $\delta\varphi(\boldsymbol{q})$ from (A2) we have

$$\langle j_i\rangle = \langle\sigma_{ik}\rangle\langle E_k\rangle + \sum_{\boldsymbol{q}}\delta\sigma_{il}(-\boldsymbol{q})\frac{B_l(\boldsymbol{q})}{\langle\sigma_{im}\rangle q_iq_m}\langle E_l\rangle q_l \tag{A4}$$

which is the basis for calculation of the effective conductivity $\sigma_{ik}^{\mathrm{eff}}$.

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